

# Adaptive Wavelet Rendering

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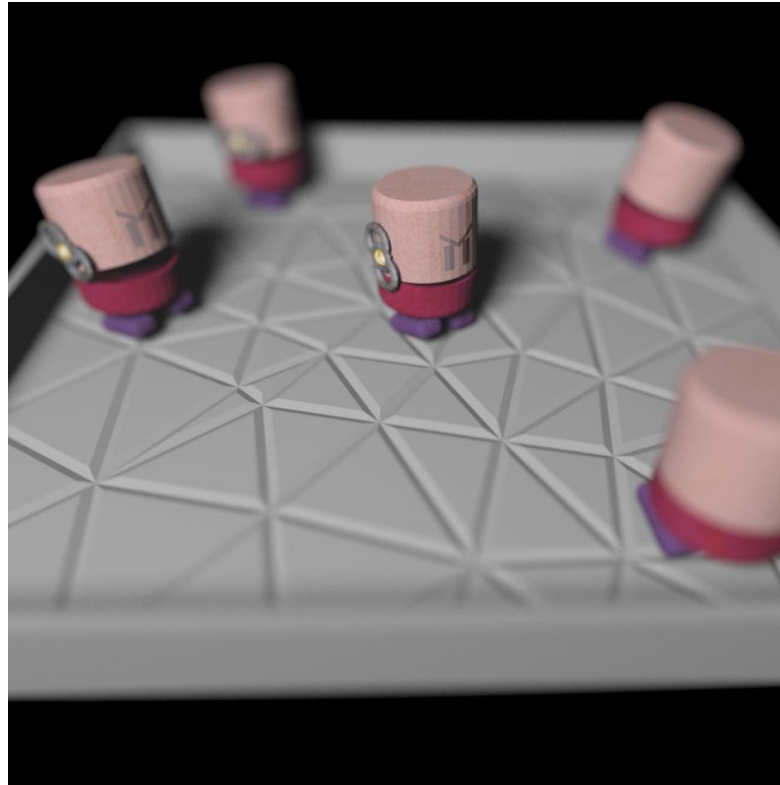
**Author: Ryan Overbeck**

**Craig Donner**

**Ravi Ramamoorthi**

**Presenter: Guillaume de Choulot**

# The Problem (combined effects)



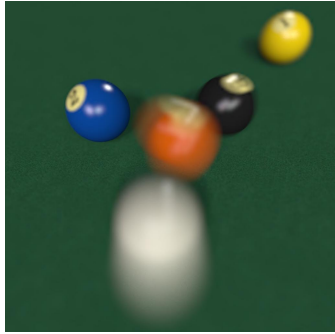
$$\text{Pixel} = \int_{\text{Pixel Area}} \int_{\text{Camera Aperture}} \int_{\text{Area Light}} \dots \rightarrow 6\text{D}$$

# General combinations of effects



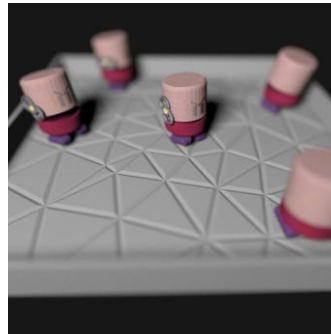
Antialiasing  
Depth of field

4D



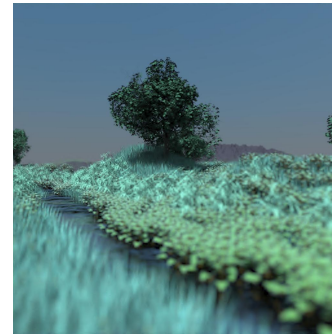
Antialiasing  
Depth of field  
Motion Blur

5D



Antialiasing  
Depth of field  
Area Lighting

6D



Antialiasing  
Depth of field  
Envir. Lighting

6D

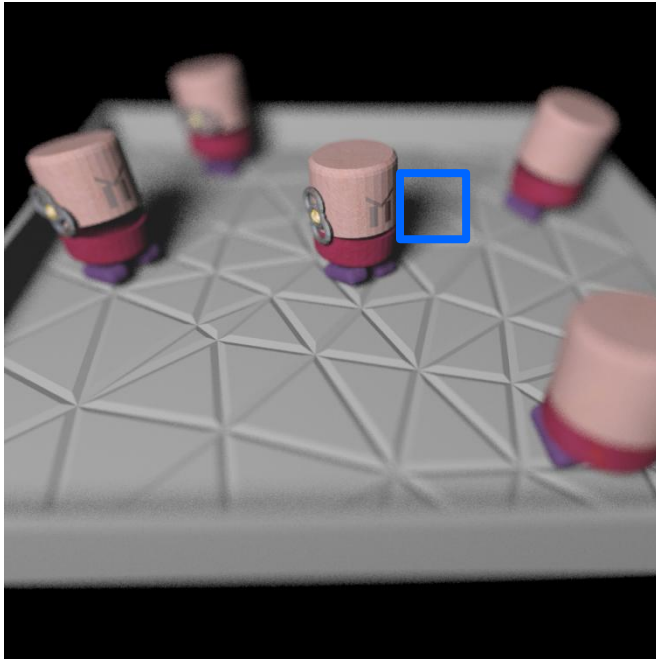


Antialiasing  
Depth of field  
Area Lighting  
1 Bounce GI

8D

# Monte Carlo Problem (1/1)

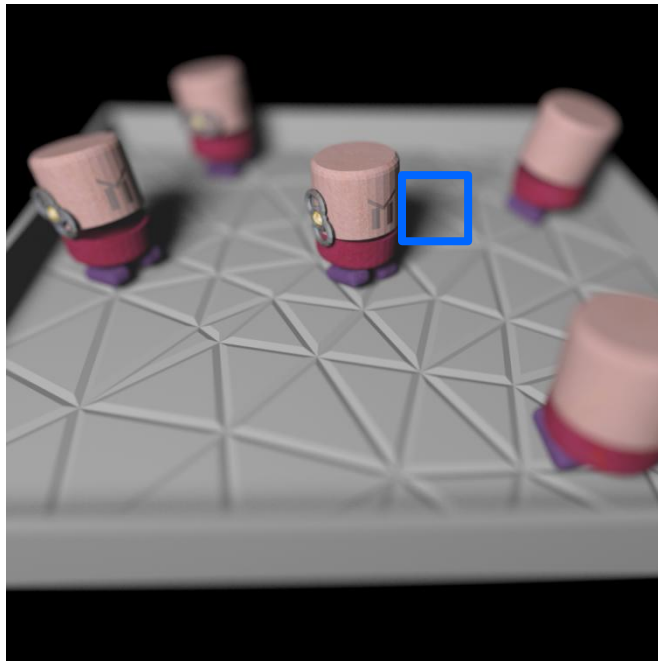
Noisy for low sample counts (smooth regions)



32 Samples Per Pixel

# Monte Carlo Problem (2/2)

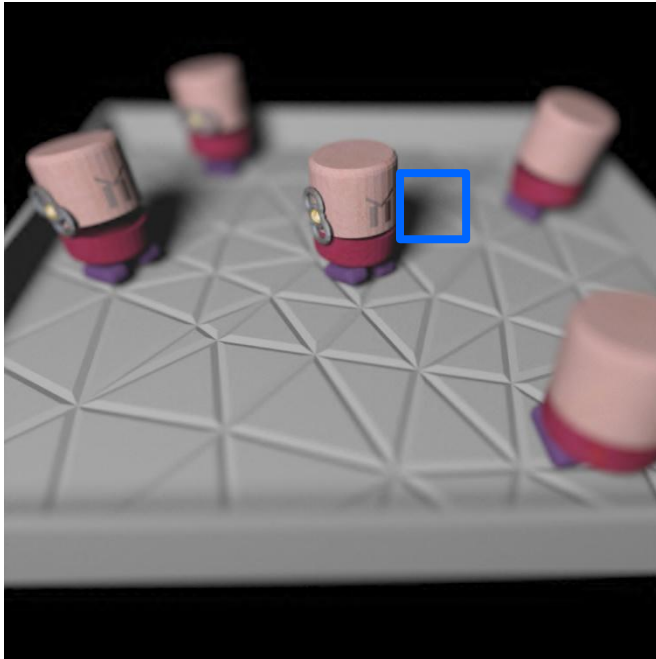
Requires hundreds to thousands of samples



512 Samples Per Pixel

# Our Solution (important sampling)

## Adaptive Wavelet Rendering



32 Samples Per Pixel  
(average)

# Features of Adaptive Wavelet Rendering

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**Low Sample Counts**

**Converges from smooth**

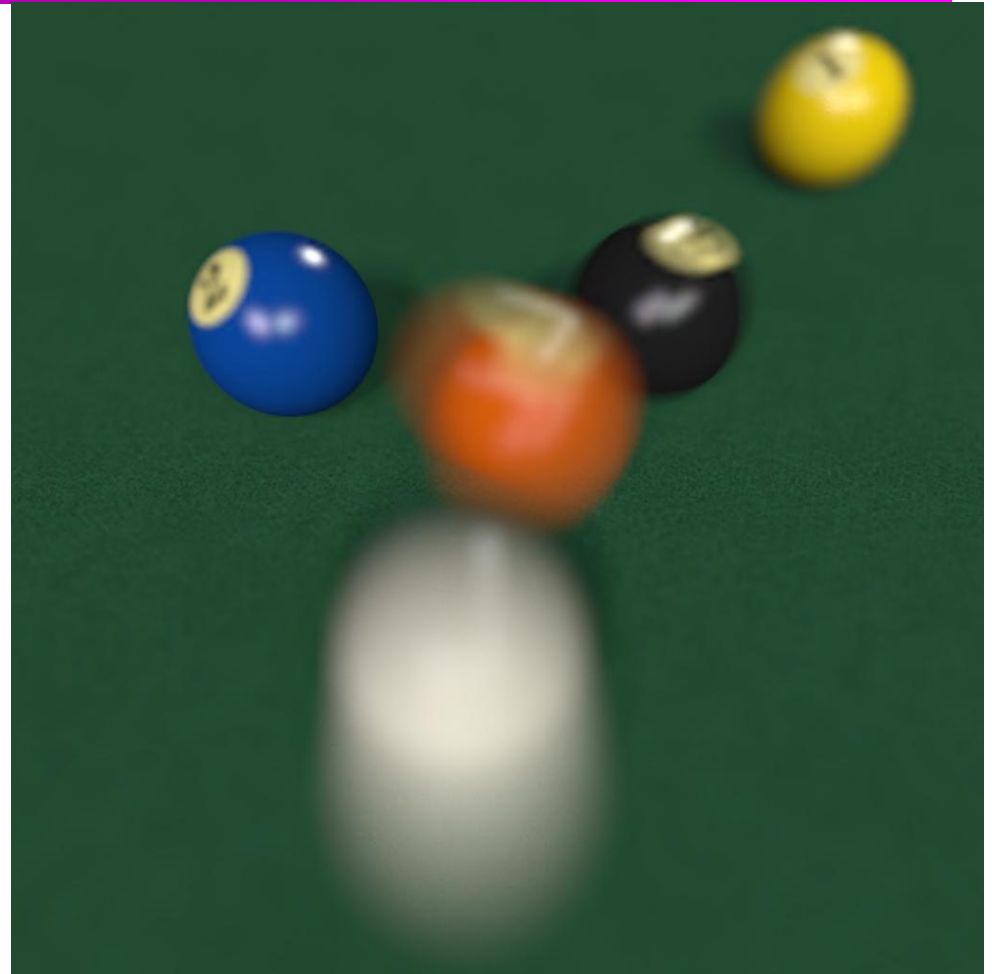
# Features of Adaptive Wavelet Rendering

## Low Sample Counts

Converges from smooth

Near Reference:

**32 samples per pixel**



Average of 32 Samples Per Pixel



# Features of Adaptive Wavelet Rendering

## Low Sample Counts

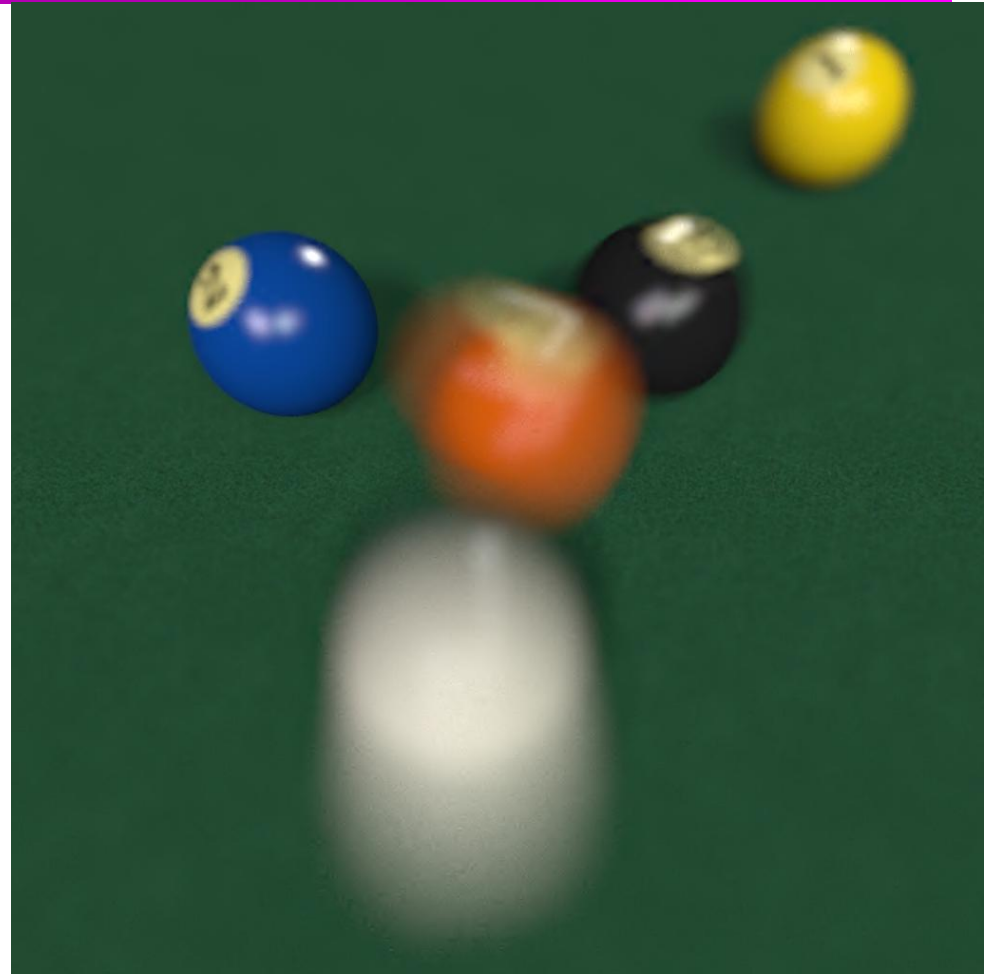
**Converges from smooth**

**Near Reference:**

**32 samples per pixel**

**Smooth Preview Quality:**

**8 samples per pixel**



Average of 8 Samples Per Pixel

# Features of Adaptive Wavelet Rendering

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## Low Sample Counts

### Efficient

**Less samples gives  
Faster render times**

# Features of Adaptive Wavelet Rendering

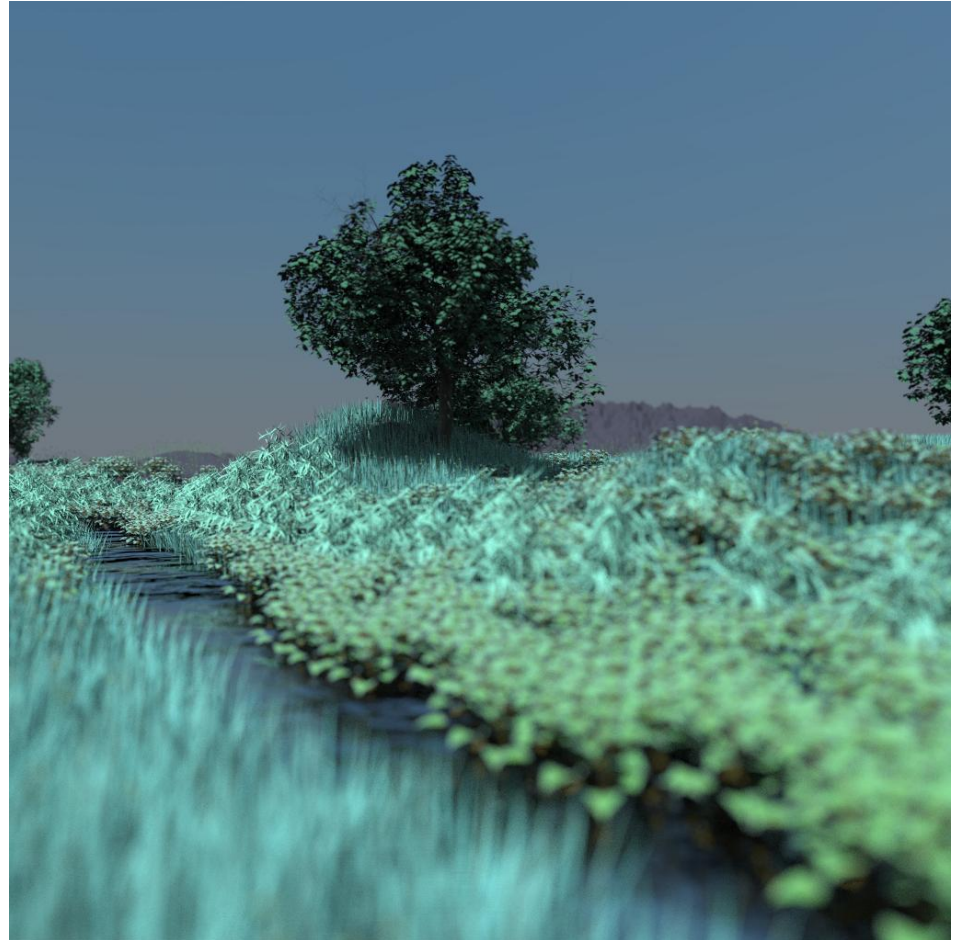
**Low Sample Counts**

**Efficient**

**Less samples gives  
Faster render times**

<b>Monte Carlo (512 spp)</b>
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>6 Hours
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> 6 Hours

Monte Carlo

512 Samples Per Pixel

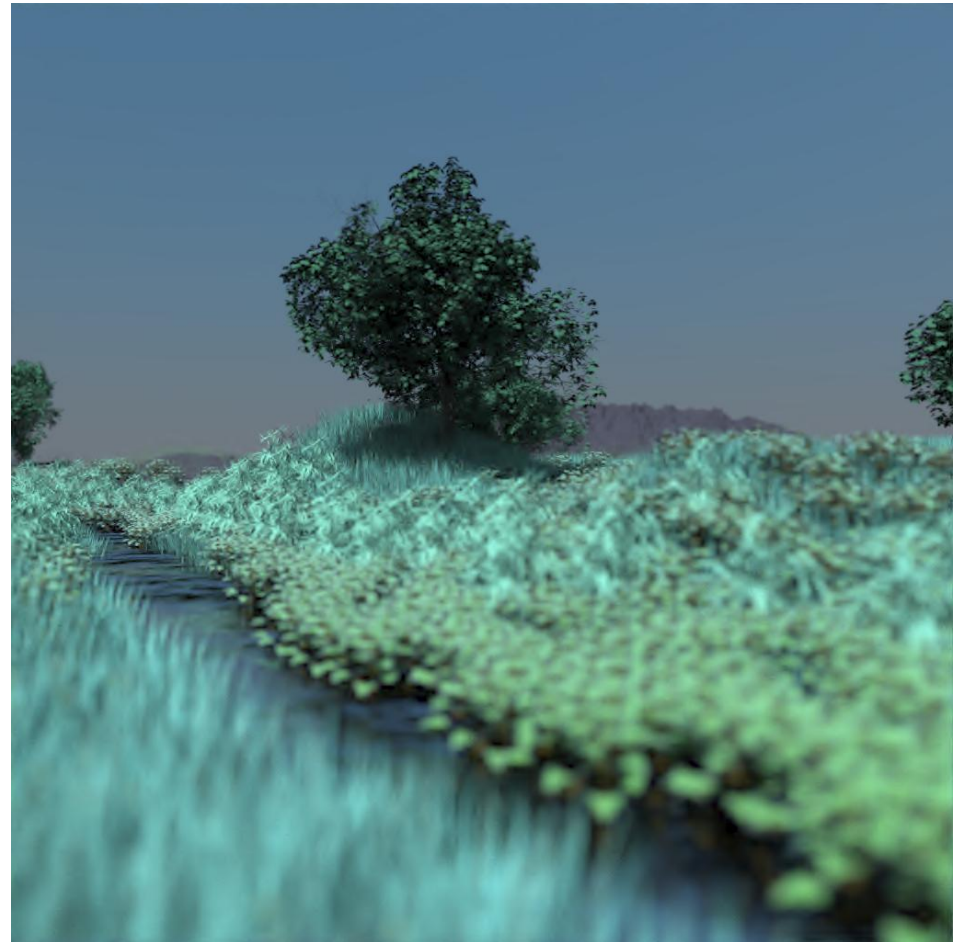
# Features of Adaptive Wavelet Rendering

**Low Sample Counts**

**Efficient**

**Less samples gives  
Faster render times**

<b>Monte Carlo (512 spp)</b>	<b>Our Method (32 spp)</b>
>6 Hours	34 minutes



**34 Minutes**

Adaptive Wavelet Rendering  
32 Samples Per Pixel (average)

# Features of Adaptive Wavelet Rendering

**Low Sample Counts**

**Efficient**

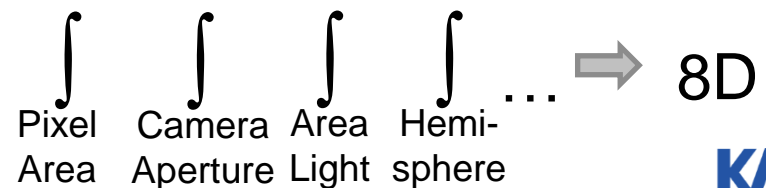
**General**

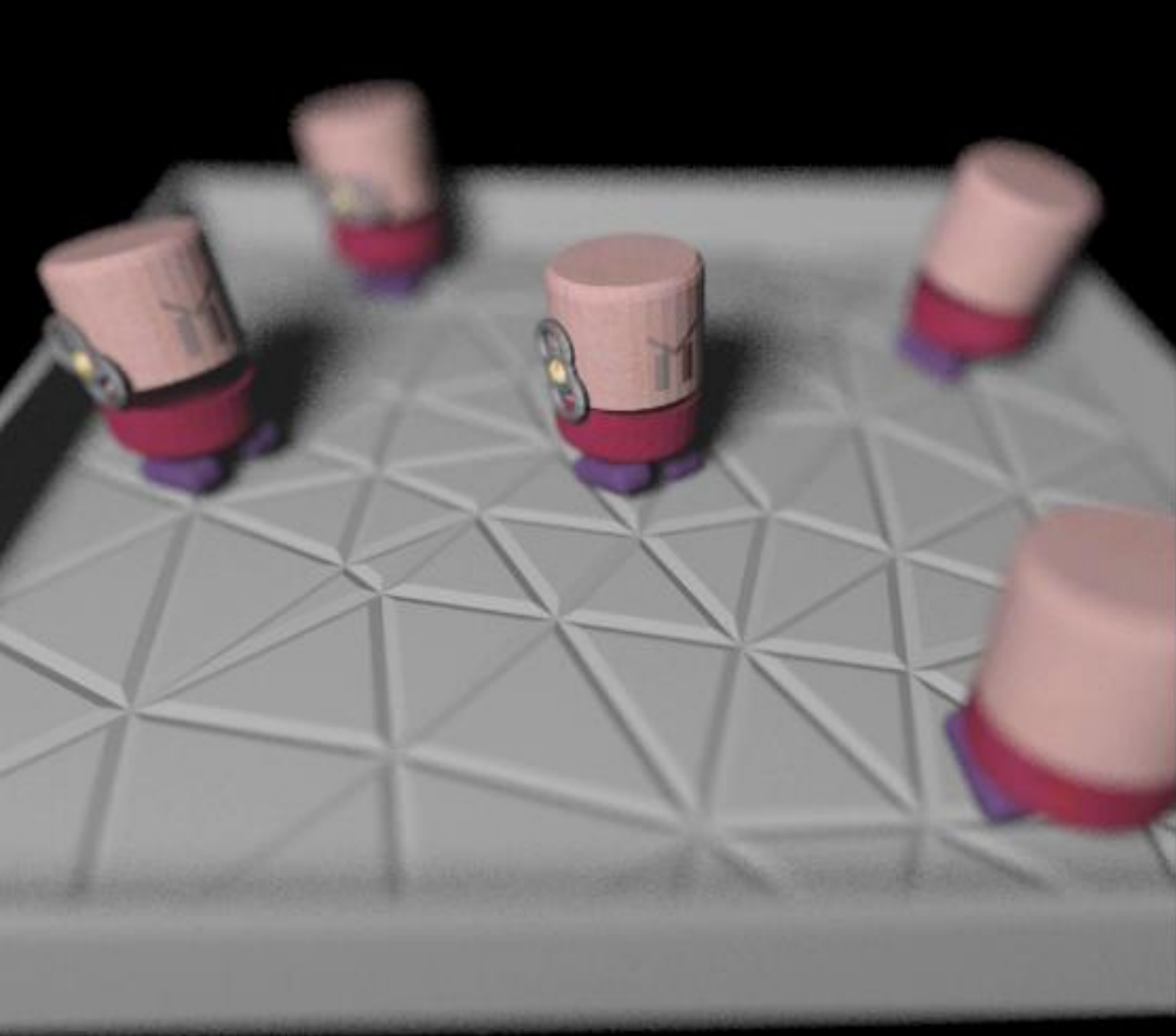
**Insensitive to problem  
dimensionality**

**General combinations  
of effects**



32 Samples per pixel (average)  
15 minutes



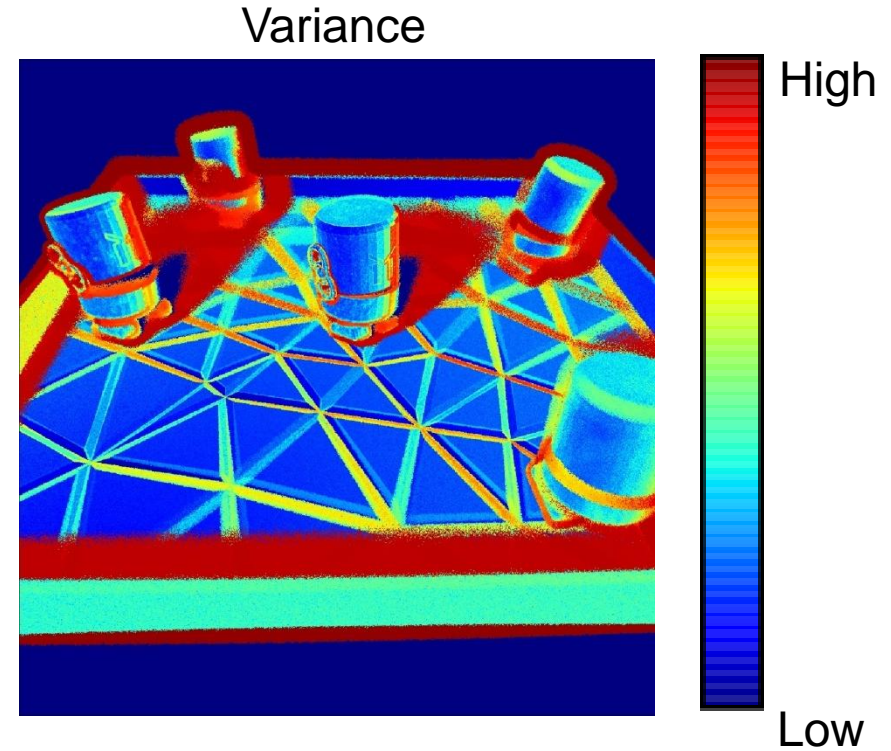
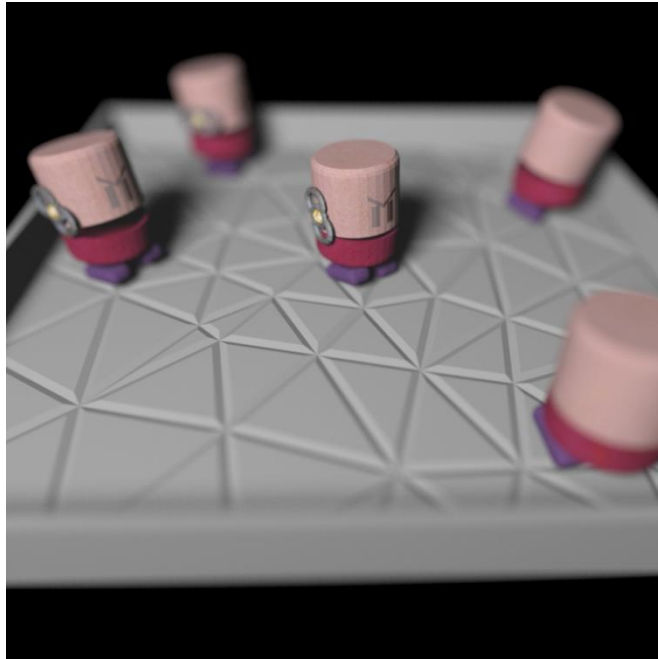


# The Insight (Important sampling)

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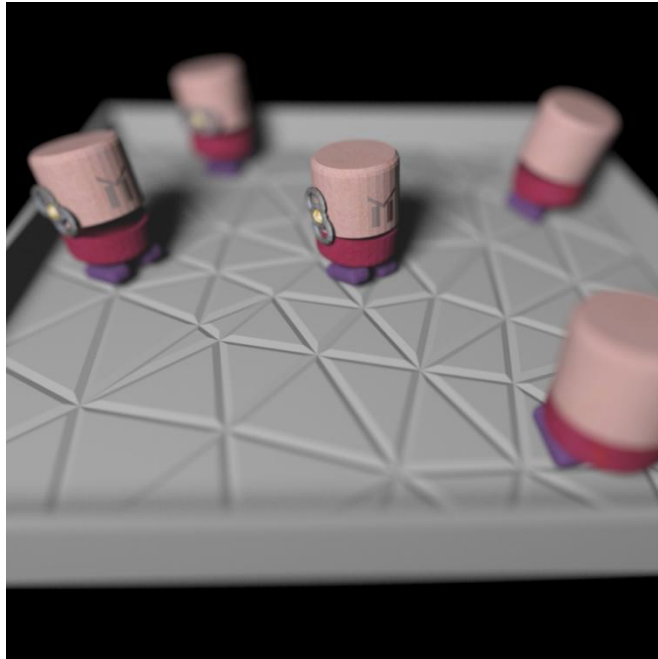
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# Variance is often local

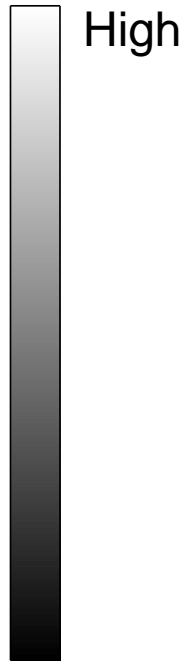
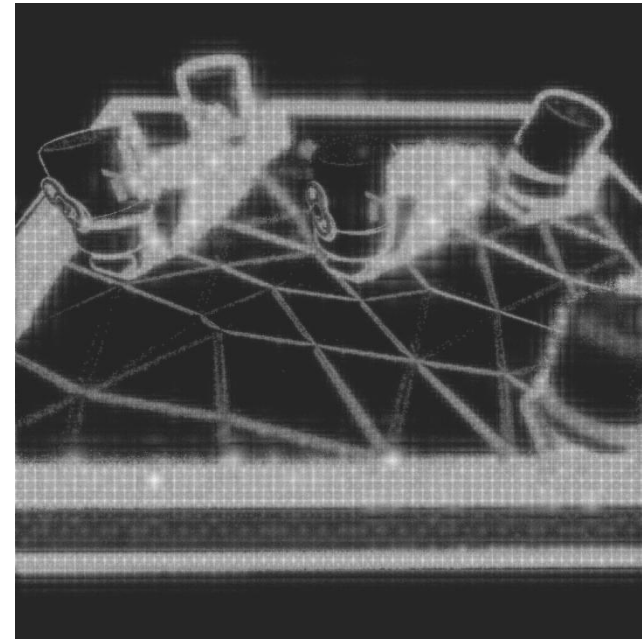




# Send more samples to high variance



Sample Count



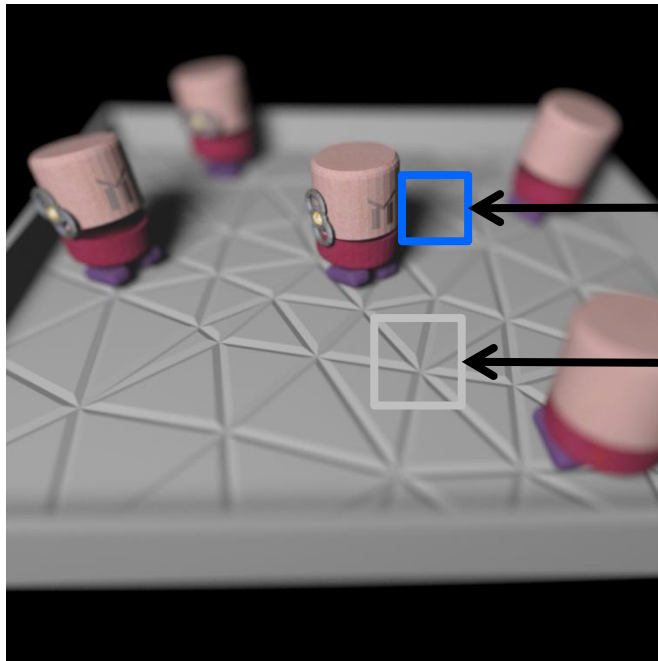
Our method

32 Samples Per Pixel  
(average)

# Two forms of variance

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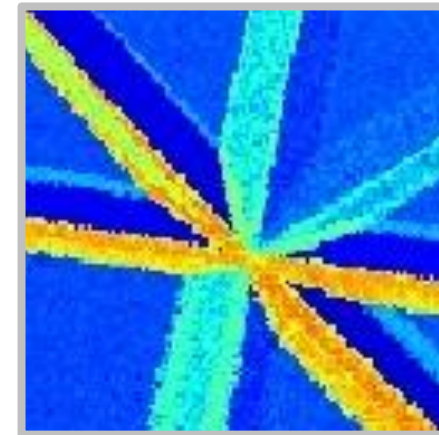
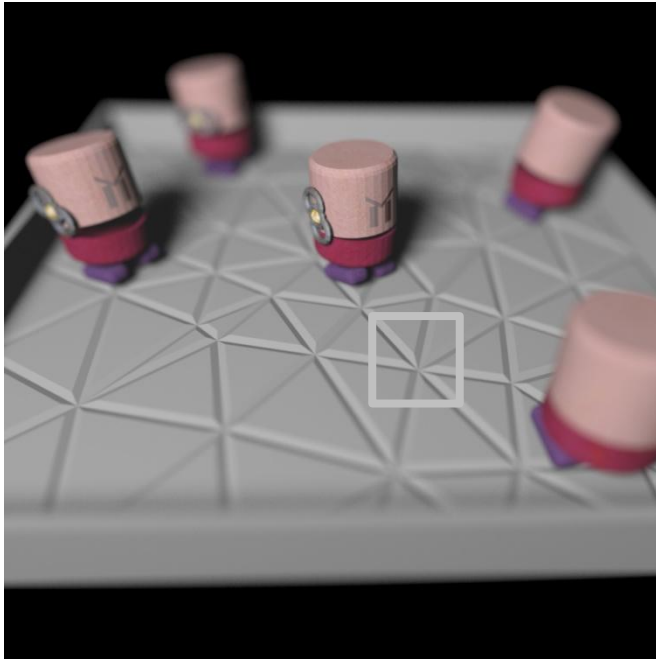
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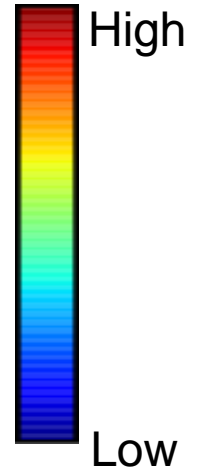
Smooth Variance

Edges

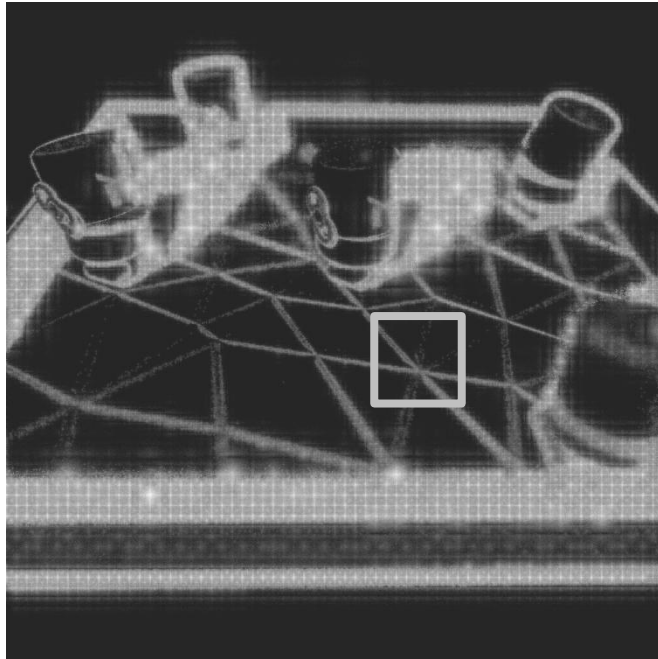
# Variance from image space: edges



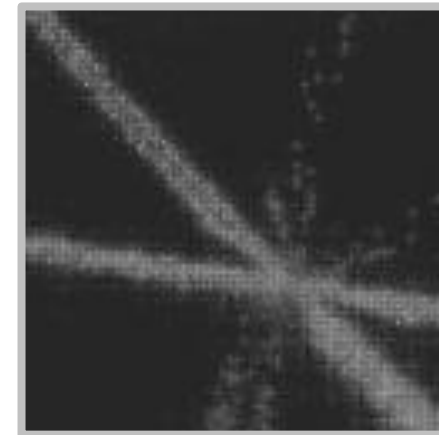
Variance



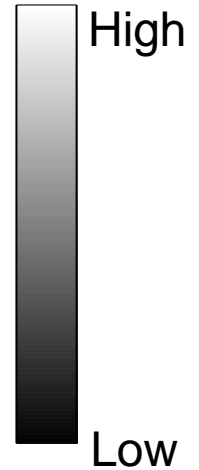
# Focused samples to image edges



Final Result

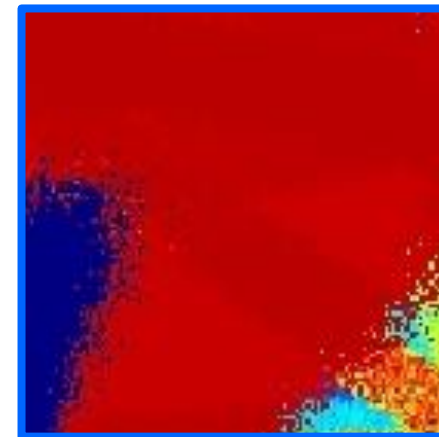
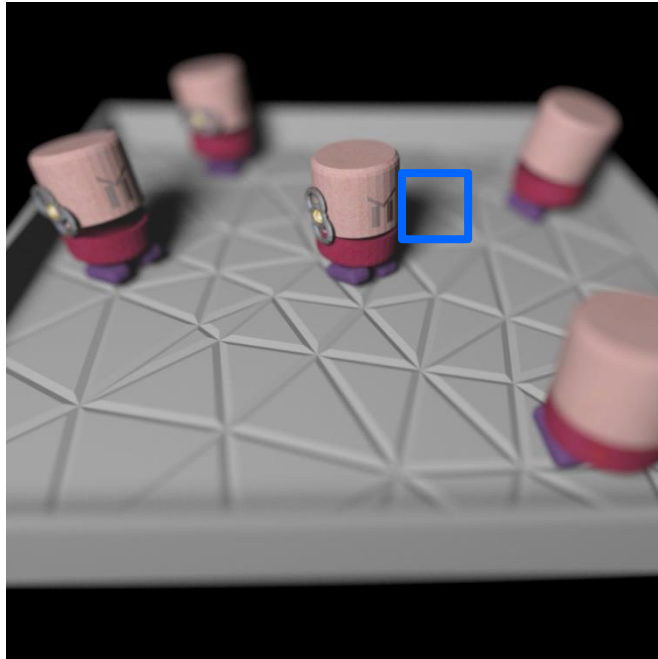


Samples



# Variance from other dims: smooth

Difficult for Monte Carlo



Variance

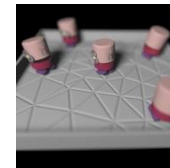
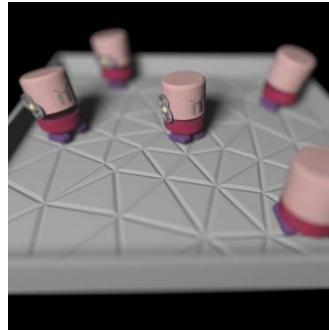
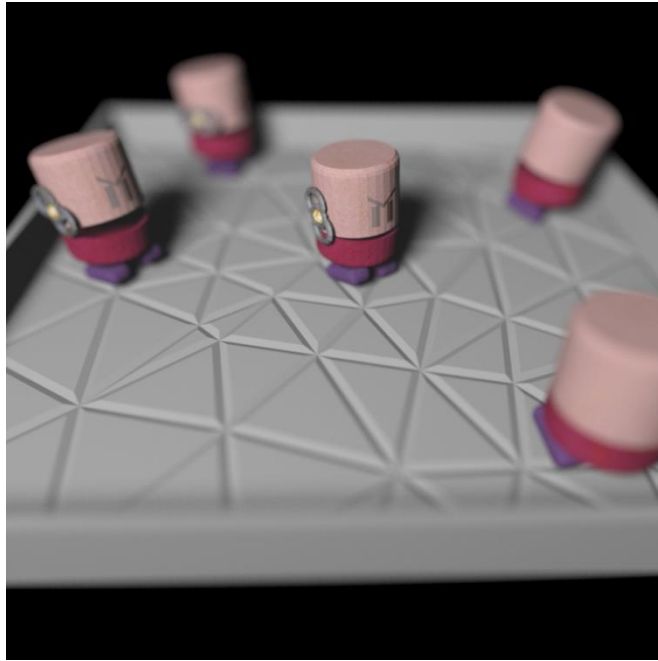
High

Low

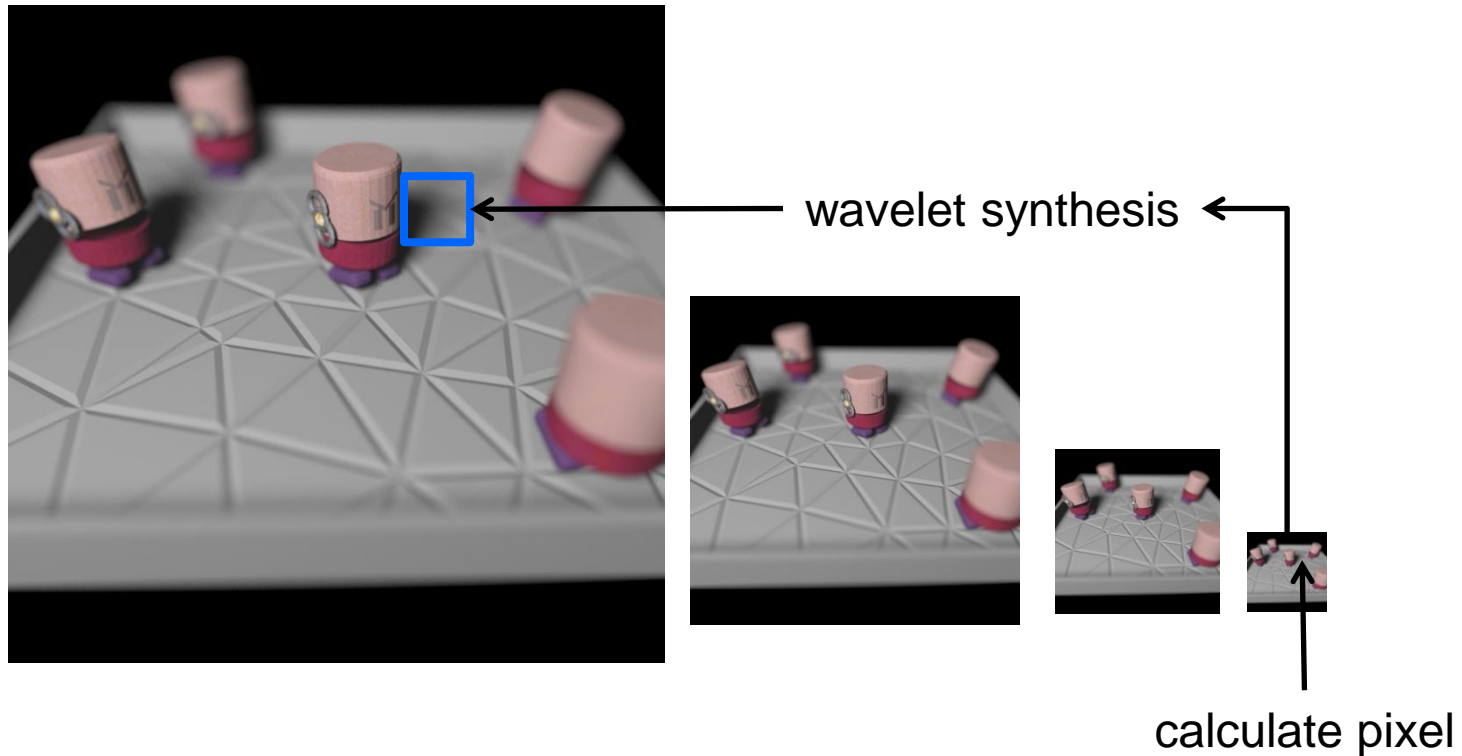
# Smooth is easier in multi-scale

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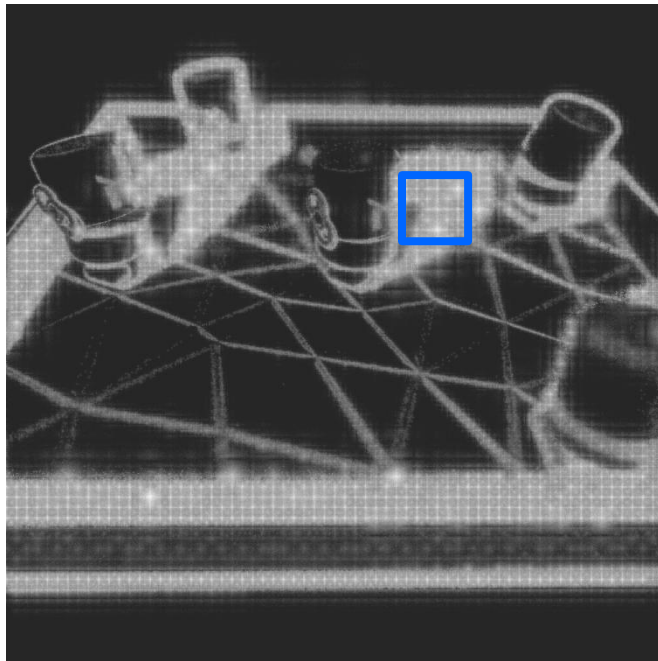
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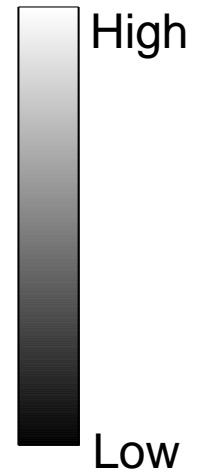
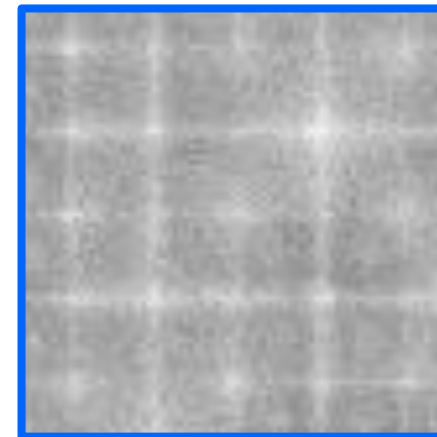
# Smooth is easier for wavelets



# Coarse Sampling of Smooth Regions



Final Result



Samples



# Algorithm Outline

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**Start: 4 Samples per Pixel**

**Adaptive Sampling**

**Reconstruction**

# Related Work

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## Adaptive sampling

**Bolin & Meyer 1998, Whitted 1980, Mitchell 1987,  
Veach and Guibas 1997, Walter et al. 2006  
(Multidim Lightcuts)**

## Multi-scale

**Keller 2001 (Hierarchical MC), Heinrich and  
Sindambiwe 1999, Guo 1998, Bala et al. 2003,  
Walter et al. 2005 (Lightcuts), Perona and Malik  
1990**

## Wavelet sampling and reconstruction

### Our method

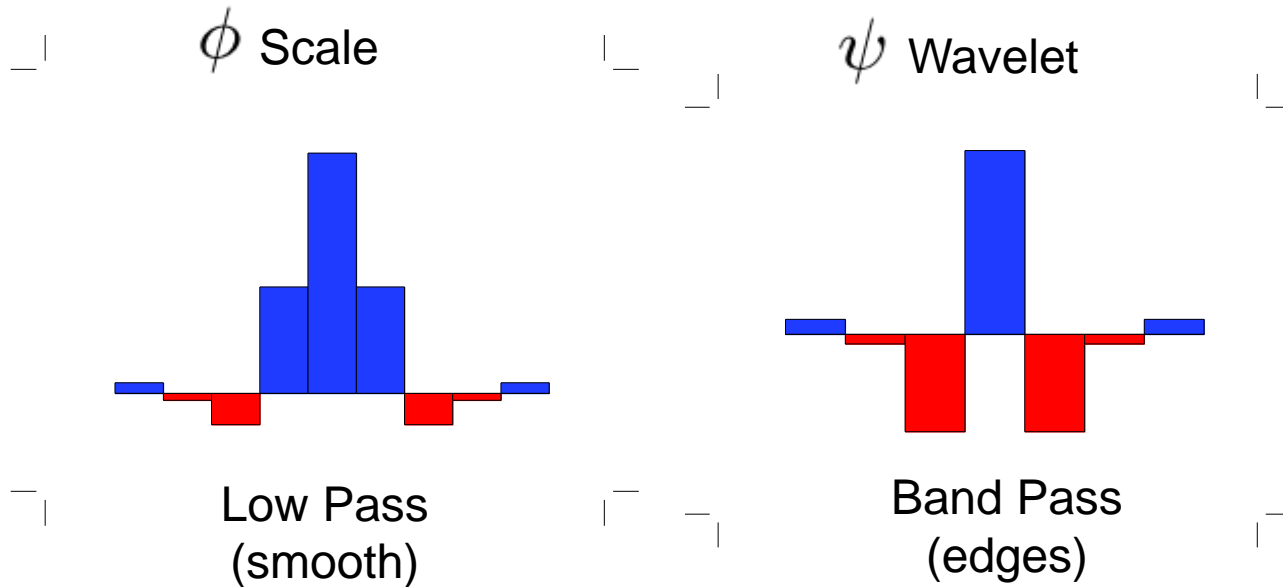
**works well for both edges and smooth regions**

# Background: Wavelets

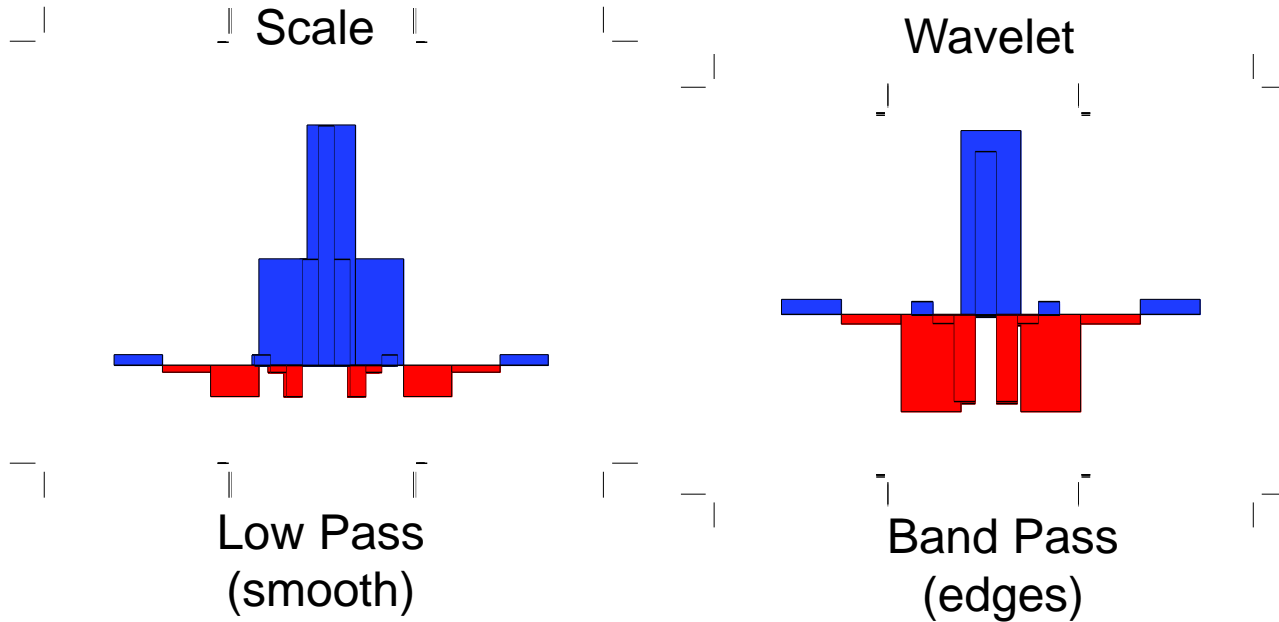
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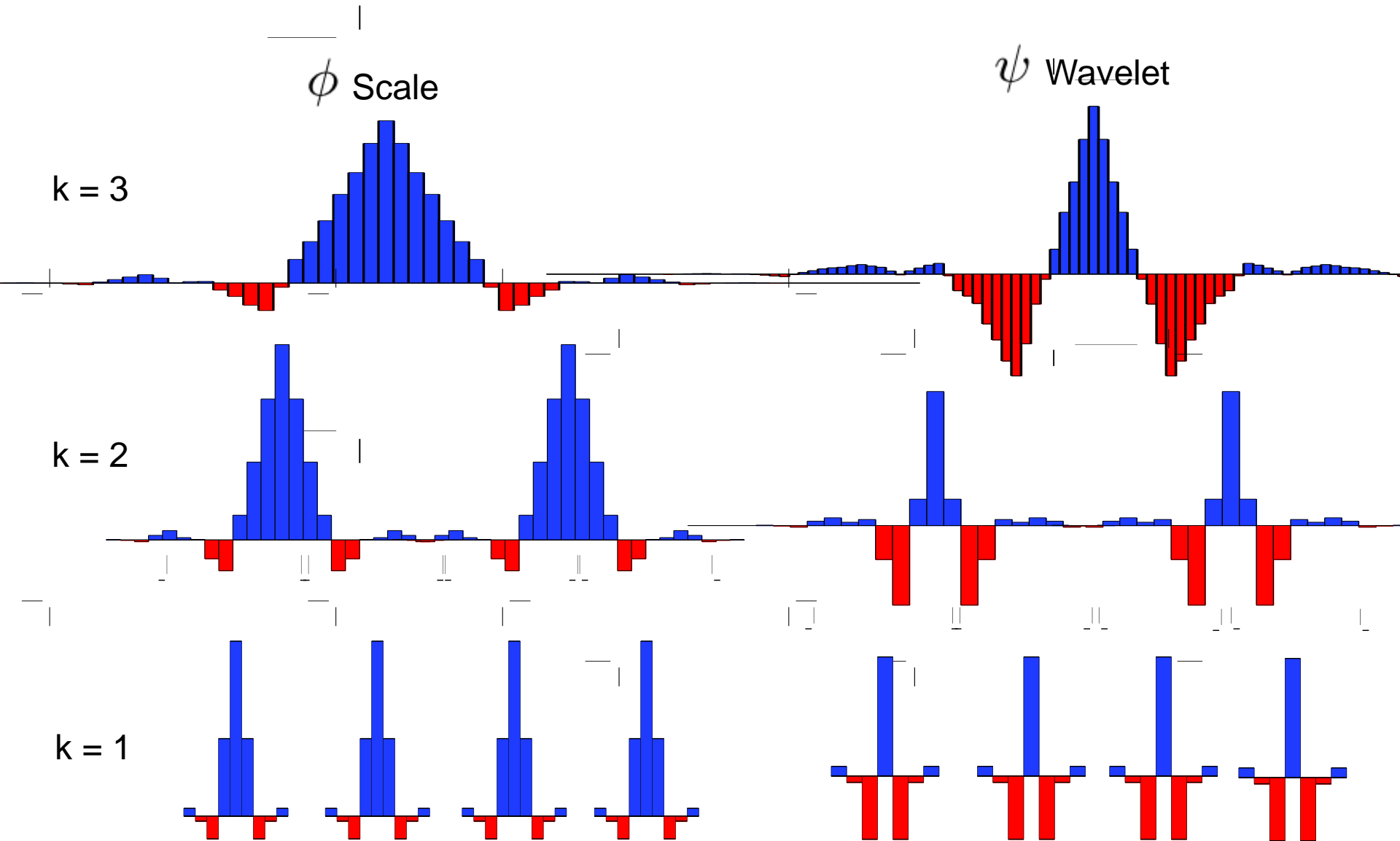
# Wavelets made of 2 functions



# Wavelet Hierarchy (1D)



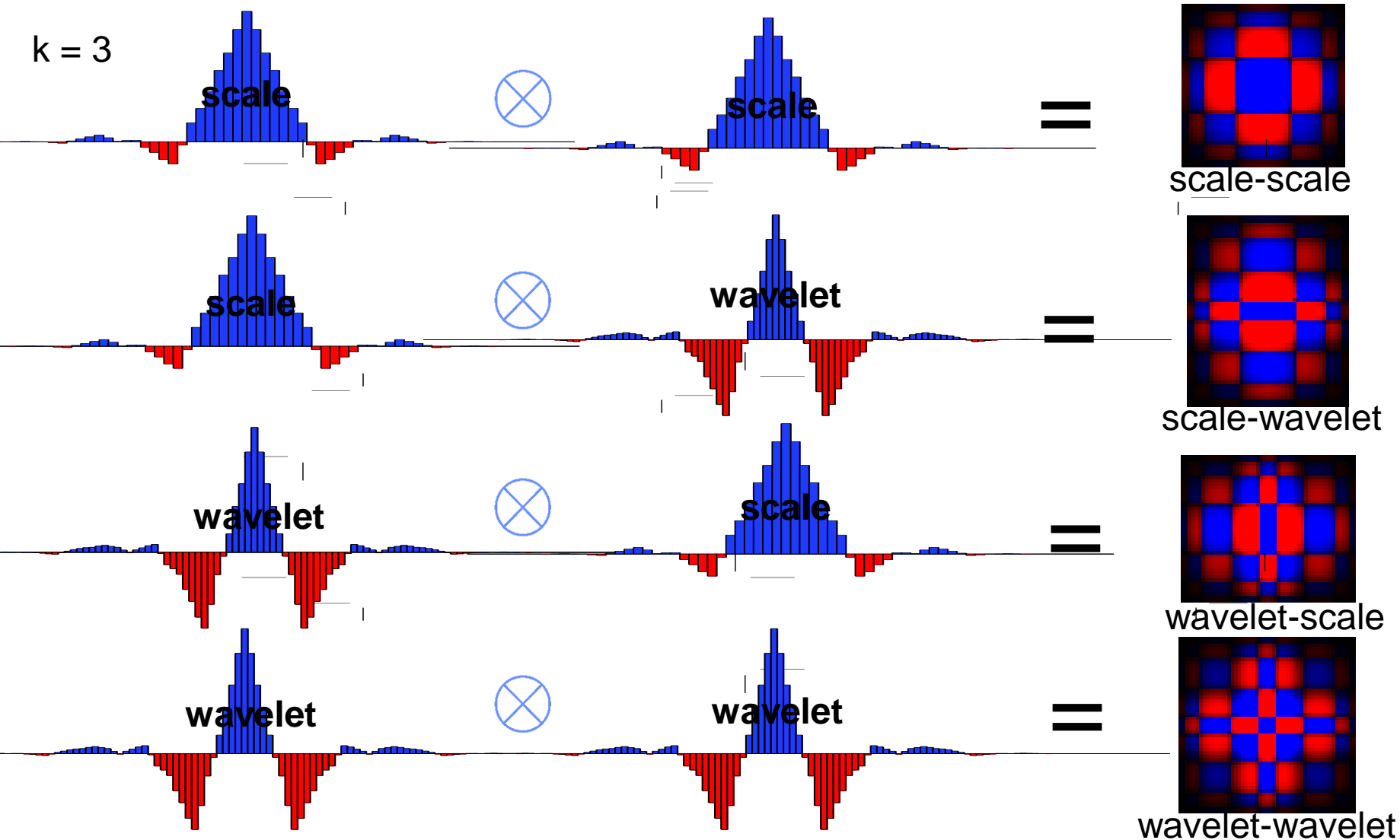
# Wavelet Hierarchy (1D)



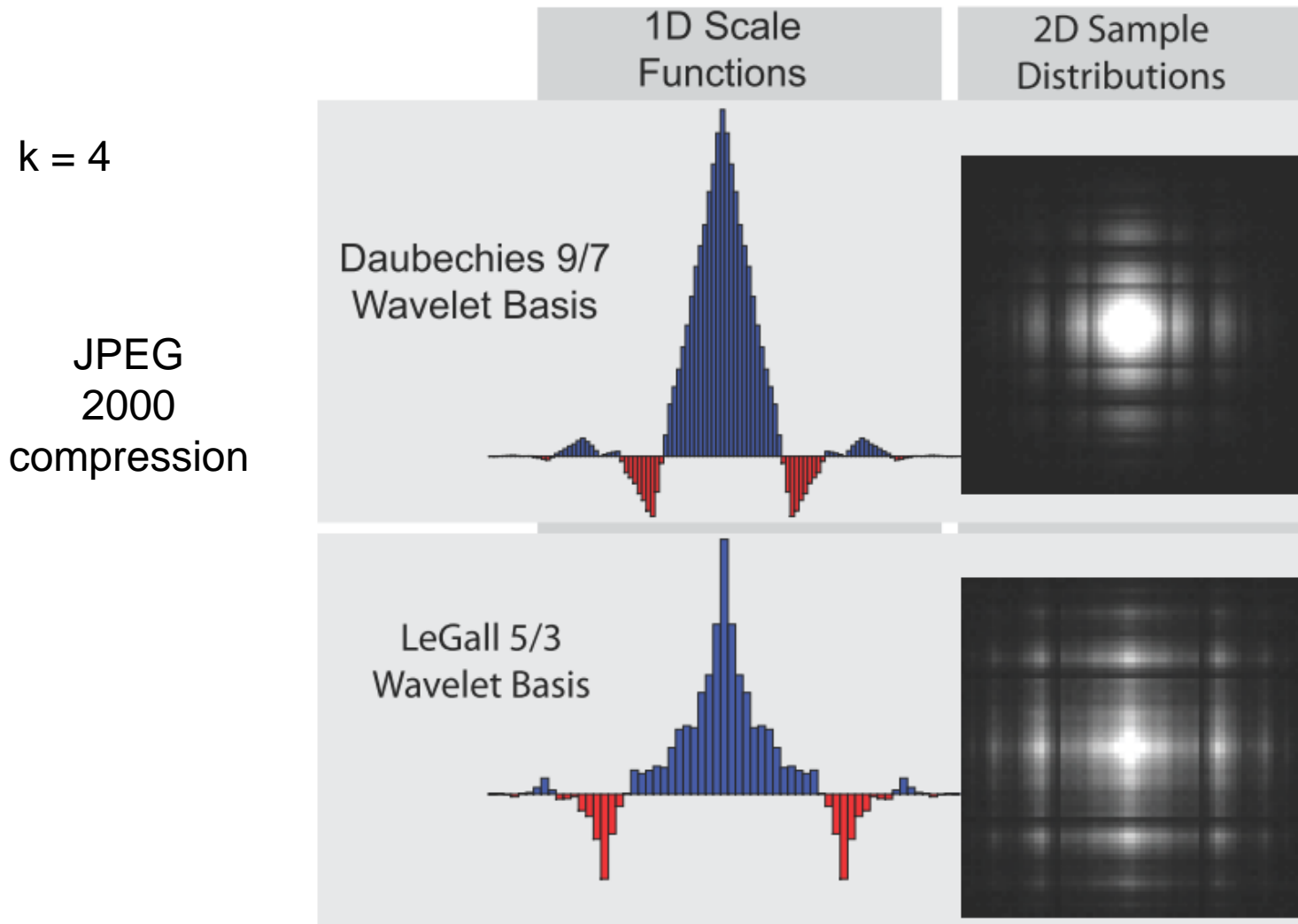
# 1D Tensor Product $\rightarrow$ 2D Basis

$$\Phi = \phi \otimes \phi^T, \Psi^0 = \phi \otimes \psi^T, \Psi^1 = \psi \otimes \phi^T, \text{ and } \Psi^2 = \psi \otimes \psi^T.$$

$k = 3$



# Wavelet used





# Wavelets Basis (& DWT)

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## Wavelets (Multi-scale basis) VS Pixel Basis

**Multi scale: coefficient/wavelet expressed in different scale**

### Discret Wavelet Transform (DWT):

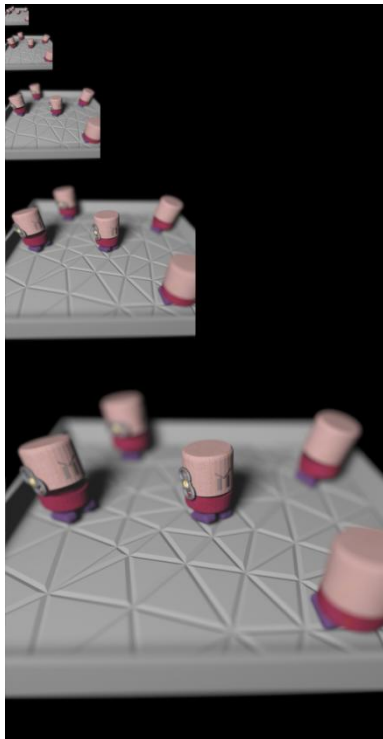
**1) Pixels  $\Rightarrow$  Wavelets coefficient (Analysis)**

$$W_{k,ij}^{\alpha} = \langle B, \Psi_{k,ij}^{\alpha} \rangle = \int \int B \cdot \Psi_{k,ij}^{\alpha} dx dy.$$

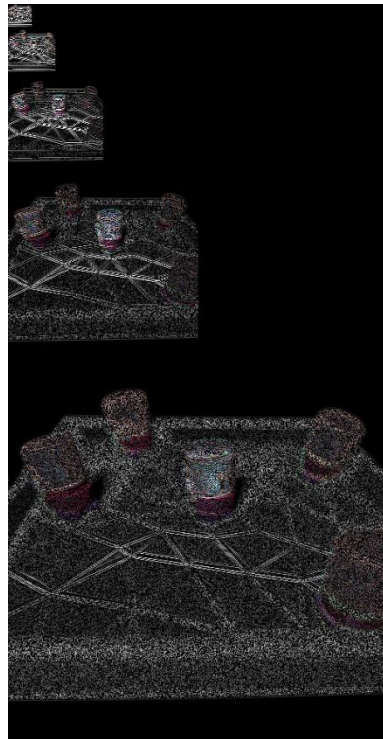
**2) Wavelets  $\Rightarrow$  Pixels (Synthesis)**

# Wavelet Hierarchy

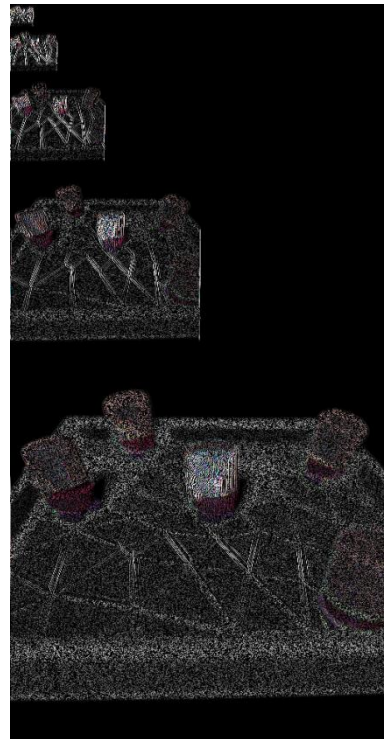
k=5  
k=4  
k=3  
k=2  
k=1



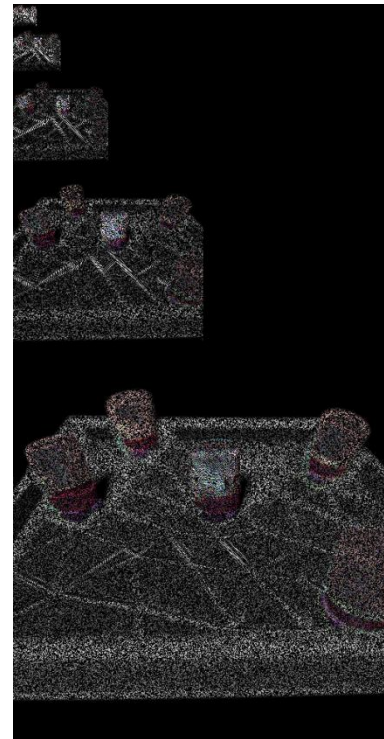
scale-scale



scale-wavelet



wavelet-scale



wavelet-wavelet

Smooth regions

Edges

$$\Phi = \phi \otimes \phi^T, \Psi^0 = \phi \otimes \psi^T, \Psi^1 = \psi \otimes \phi^T, \text{ and } \Psi^2 = \psi \otimes \psi^T.$$

# III) Algorithm Outline

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**0) Start: 4 Samples per Pixel (skipped)**

**1) Adaptive Sampling**

**2) Reconstruction**

# 1) Adaptive Sampling

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Insert all scale coefficients into a priority queue  $S_{k,ij} = \langle B, \Phi_{k,ij} \rangle = \int \int B \cdot \Phi_{k,ij} dx dy,$

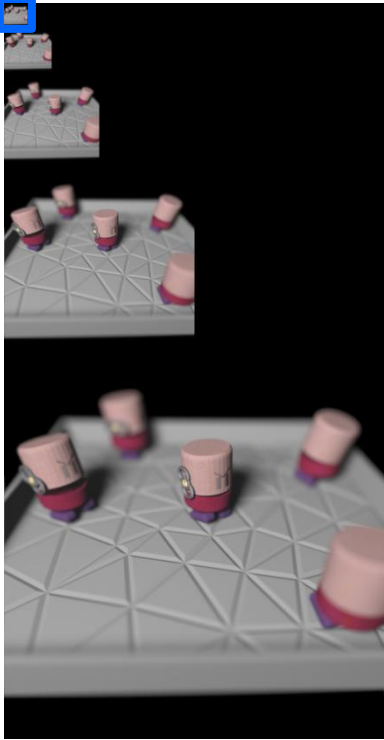
While more samples:

Send samples to highest priority coefficient

Update priority queue

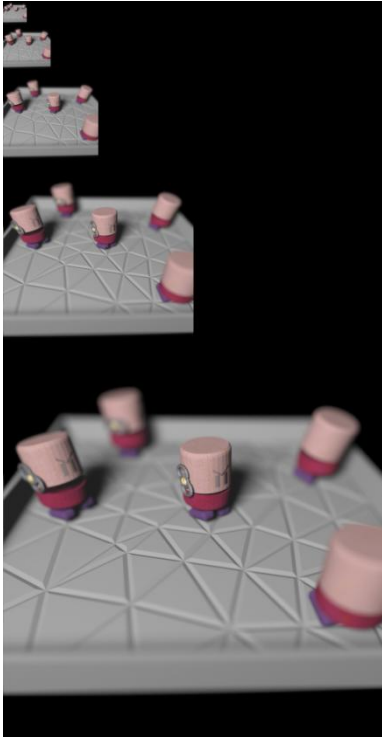
**The problem:**

**How to compute priority for each scale coefficient?**

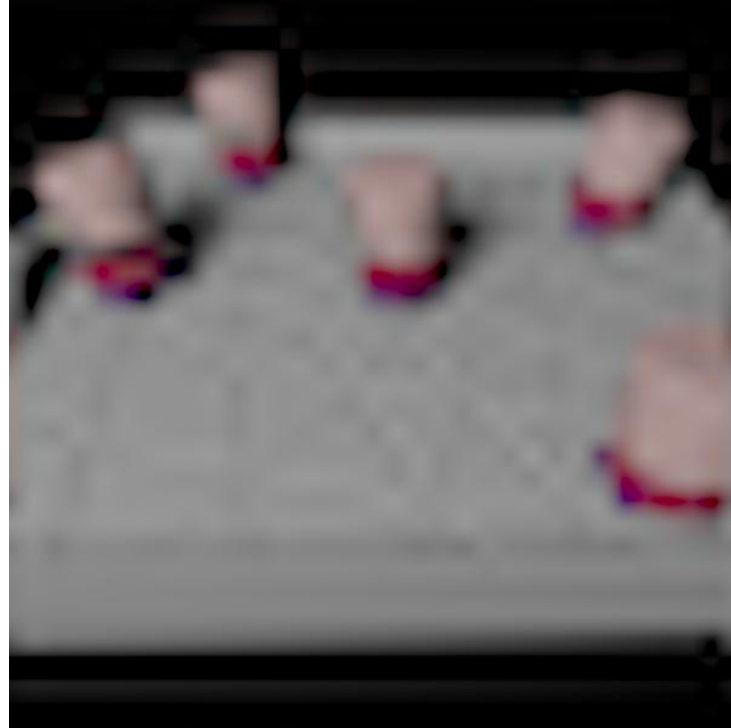


scale-scale





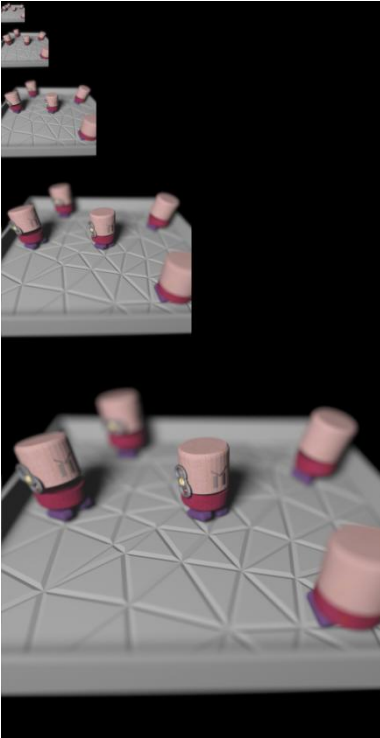
scale-scale



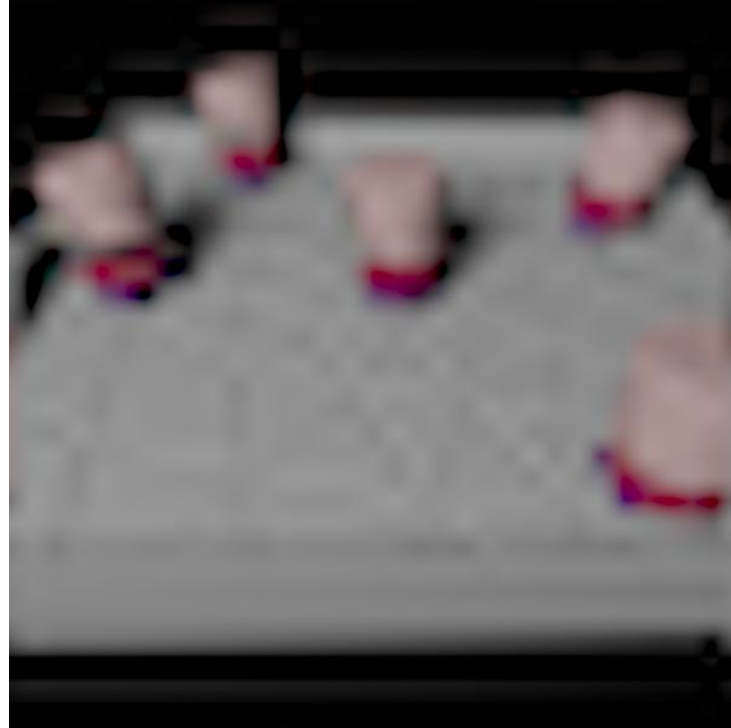
# Wavelet synthesis is smoothing

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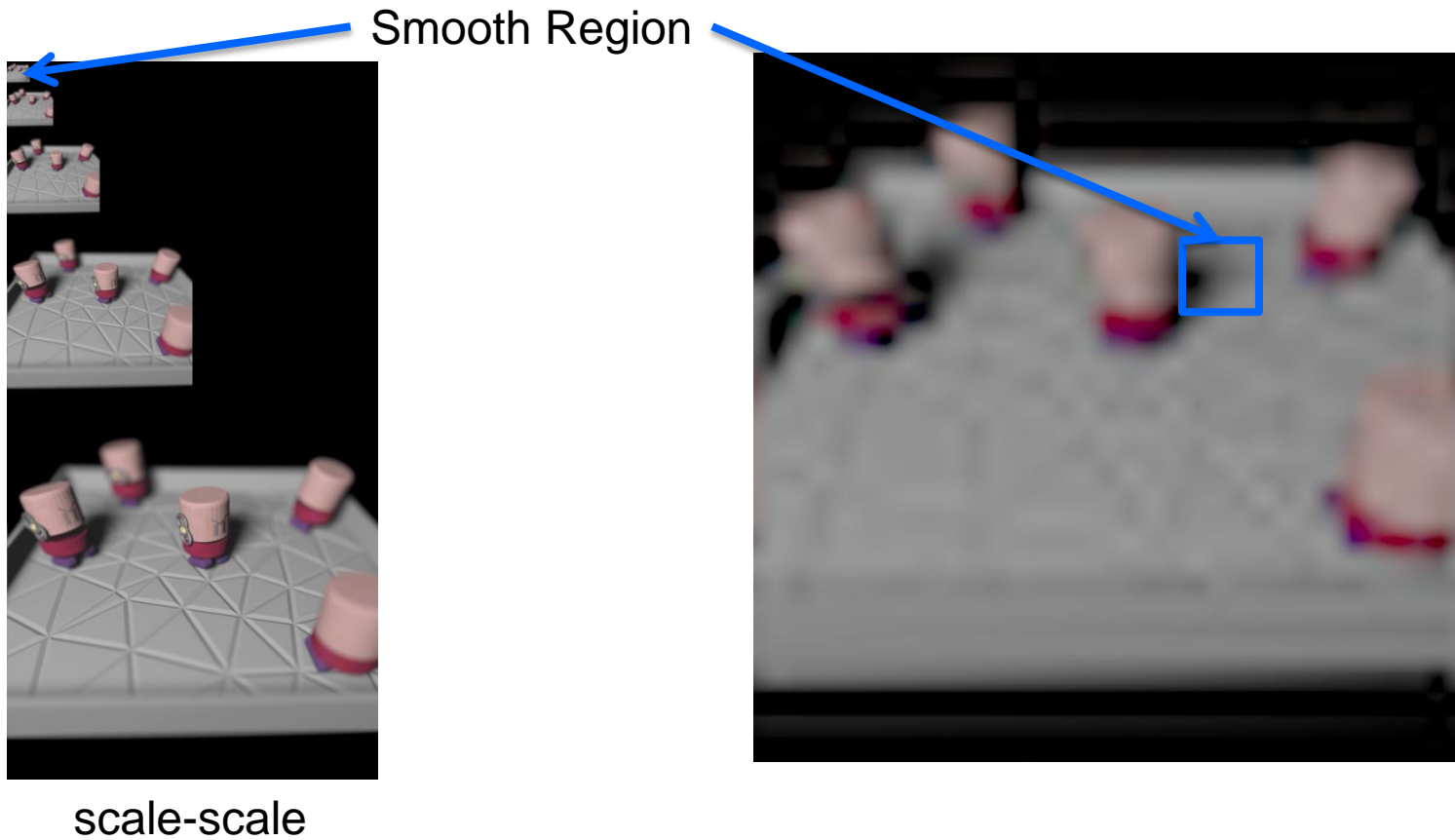
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scale-scale

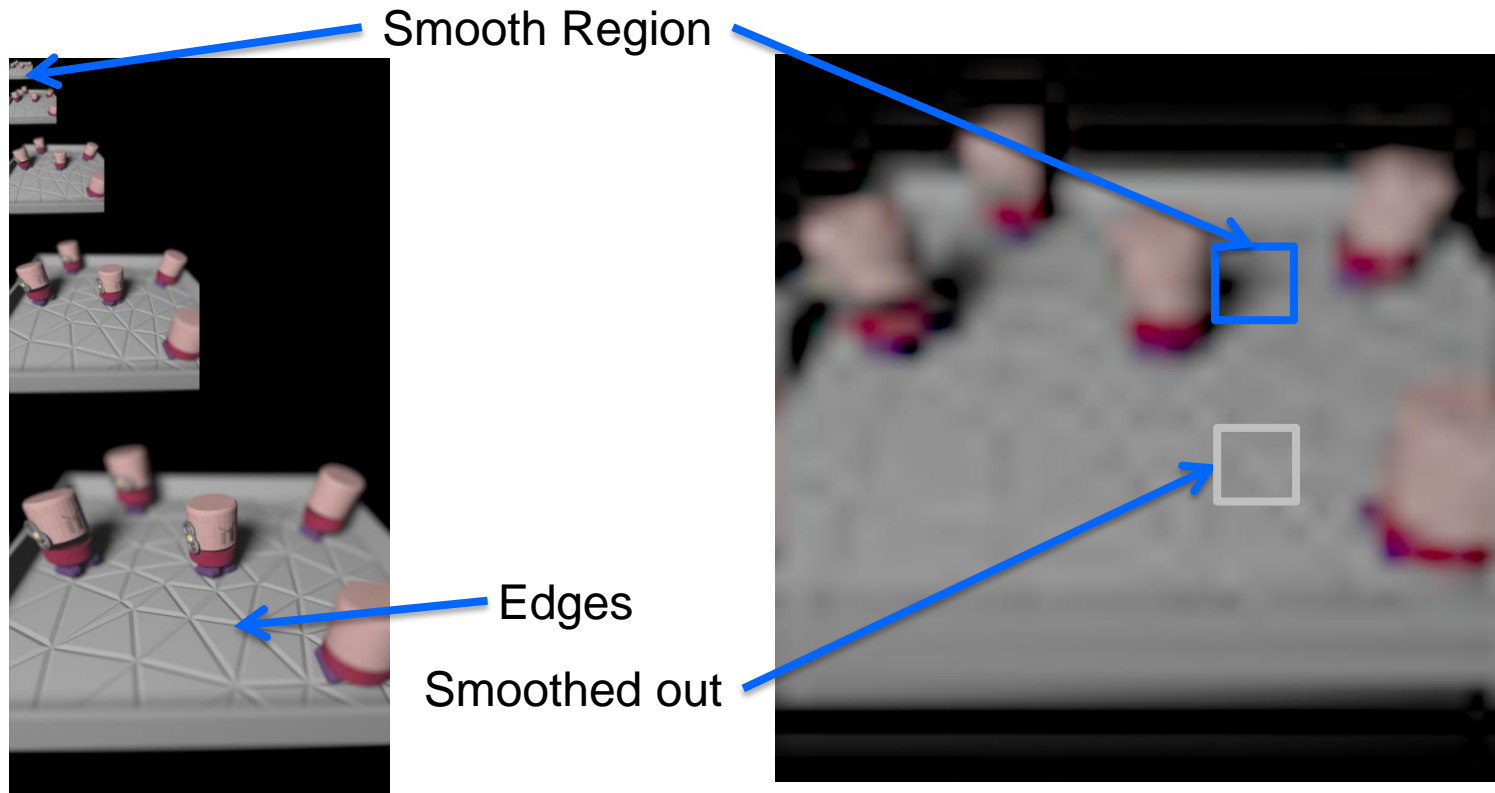


# Coarse scale captures smoothness





# Edges are in the fine scale



scale-scale

# Adaptive Sampling

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## Goals:

**In smooth regions, more samples to coarse coefficients**

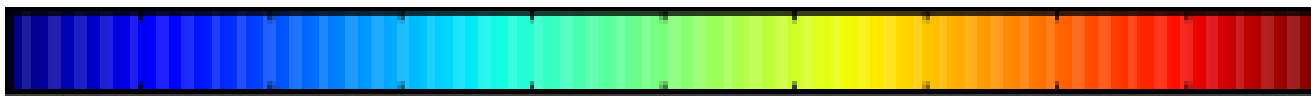
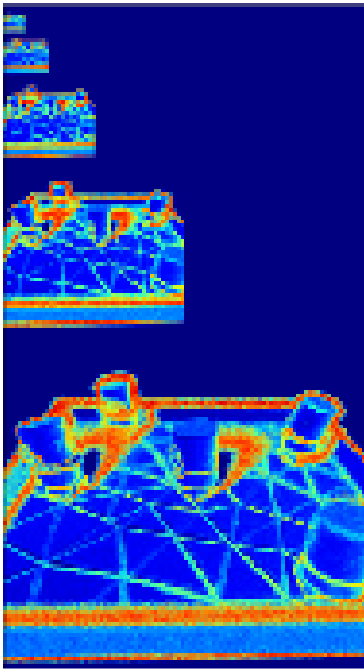
**Near edges, more samples to fine coefficients**

## Solution:

**Compute priority based on variance and smoothness**

# Start with Scale Coefficients' Variance

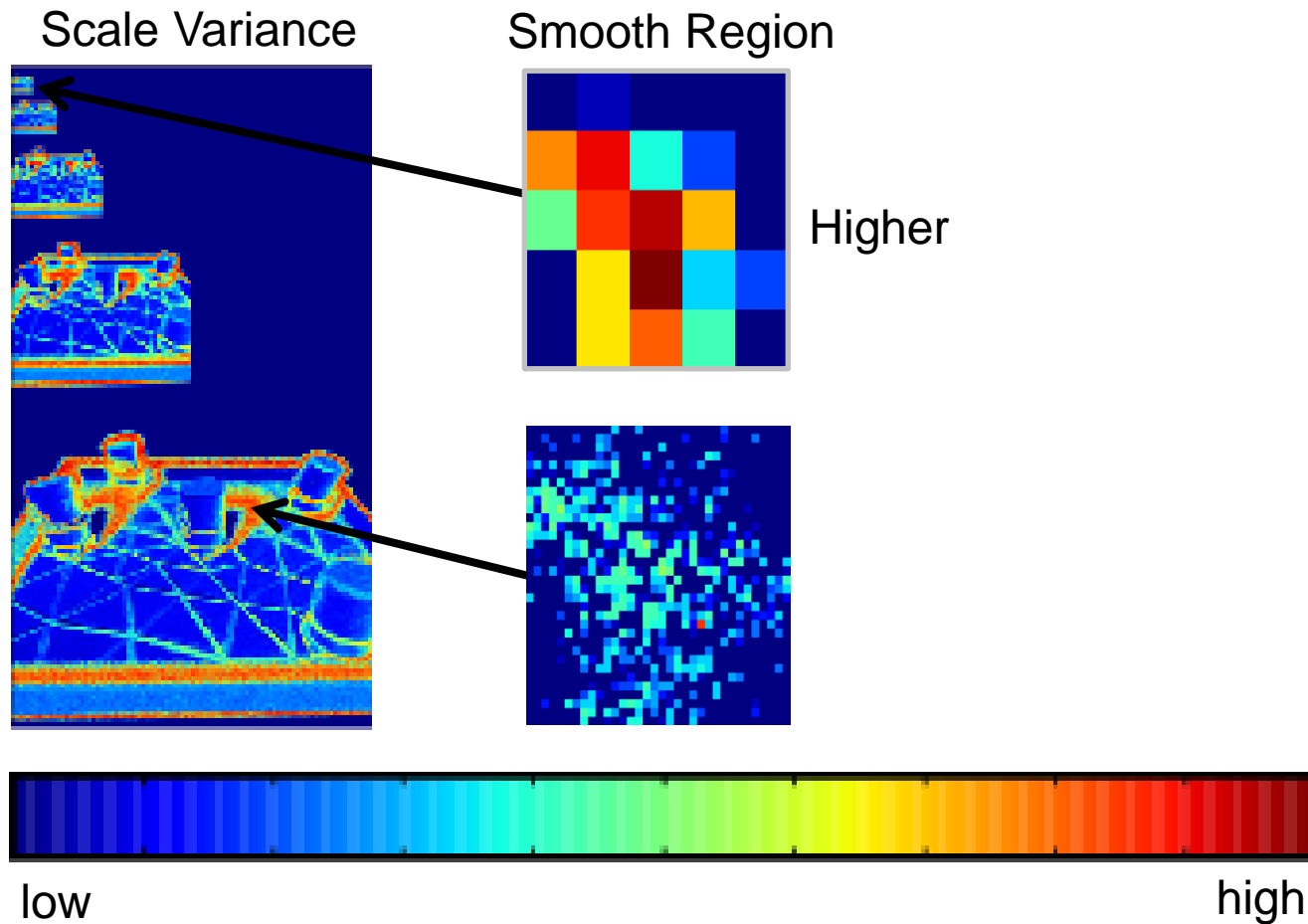
Scale Variance



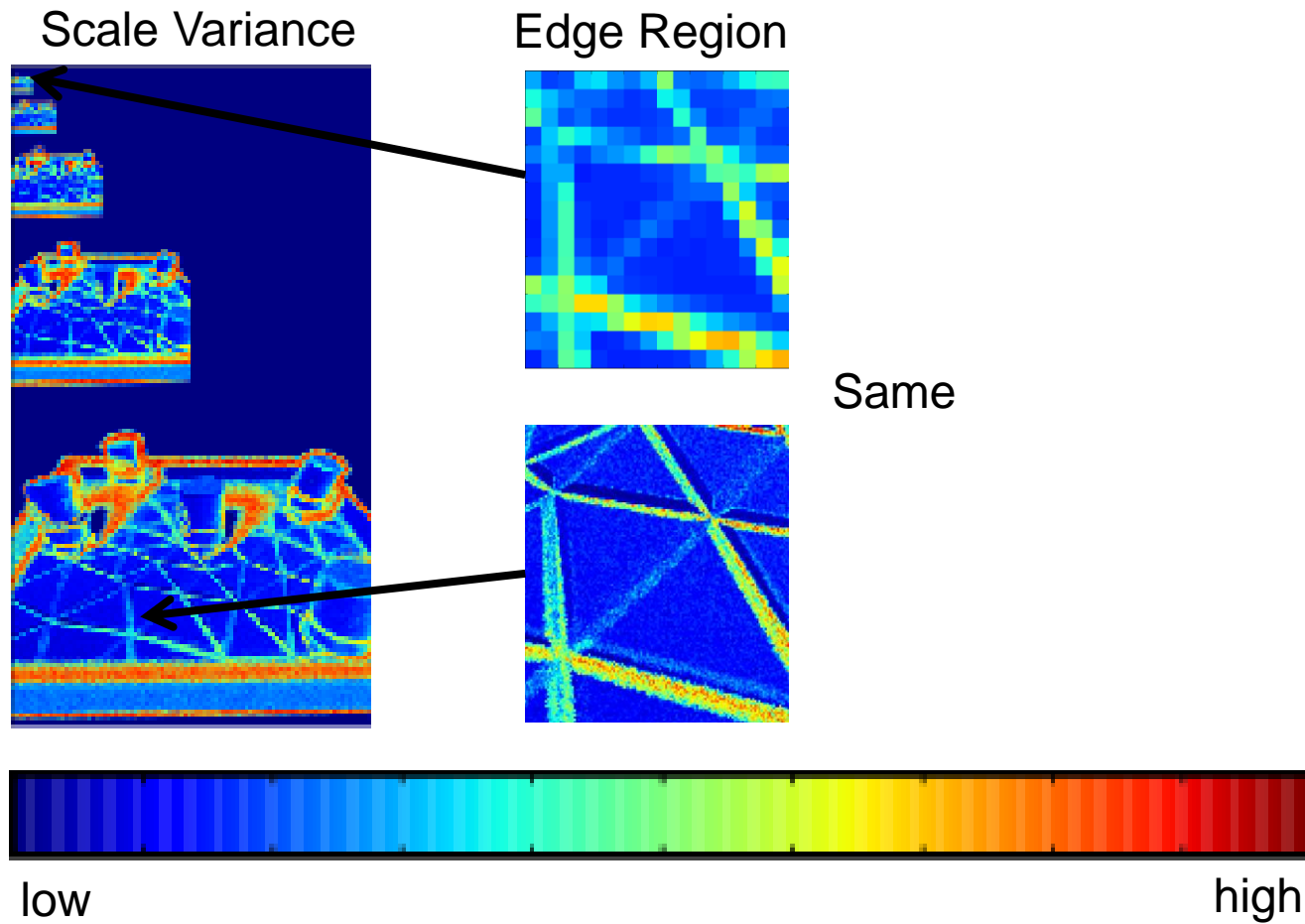
low

high

# Smooth variance grows fine to coarse

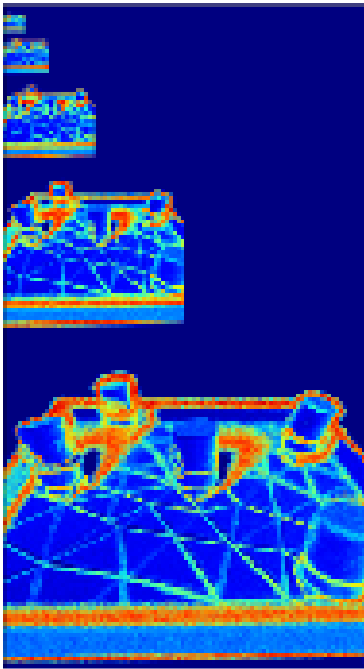


# Edge variance stays the same

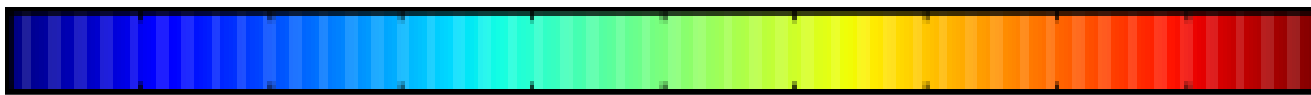
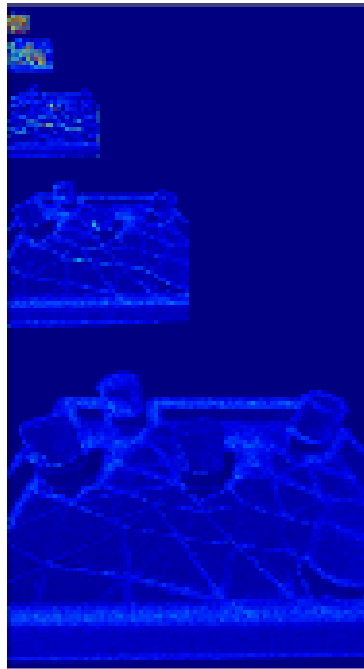


# Squared wavelet magnitudes

Scale Variance



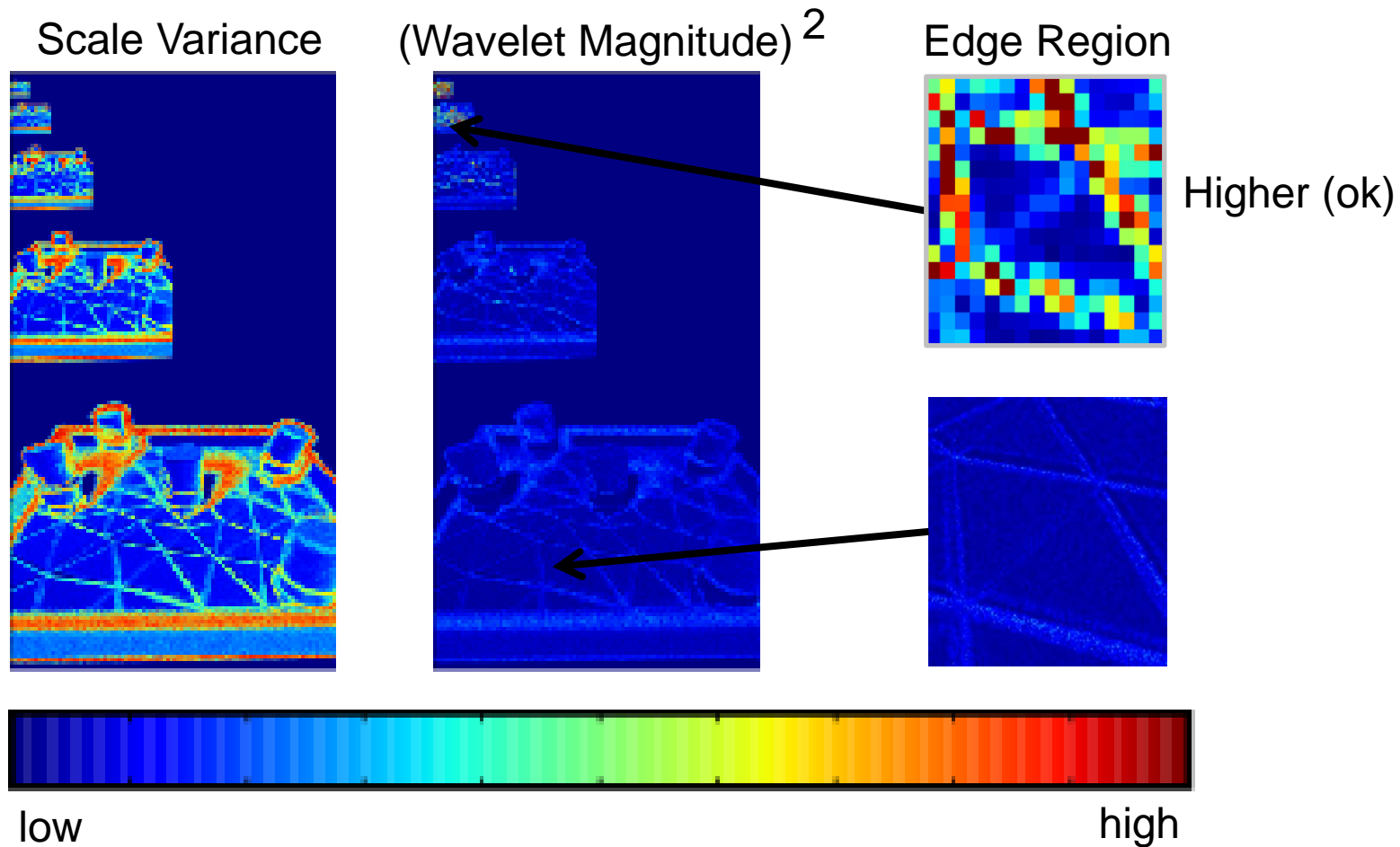
(Wavelet Magnitude)<sup>2</sup>



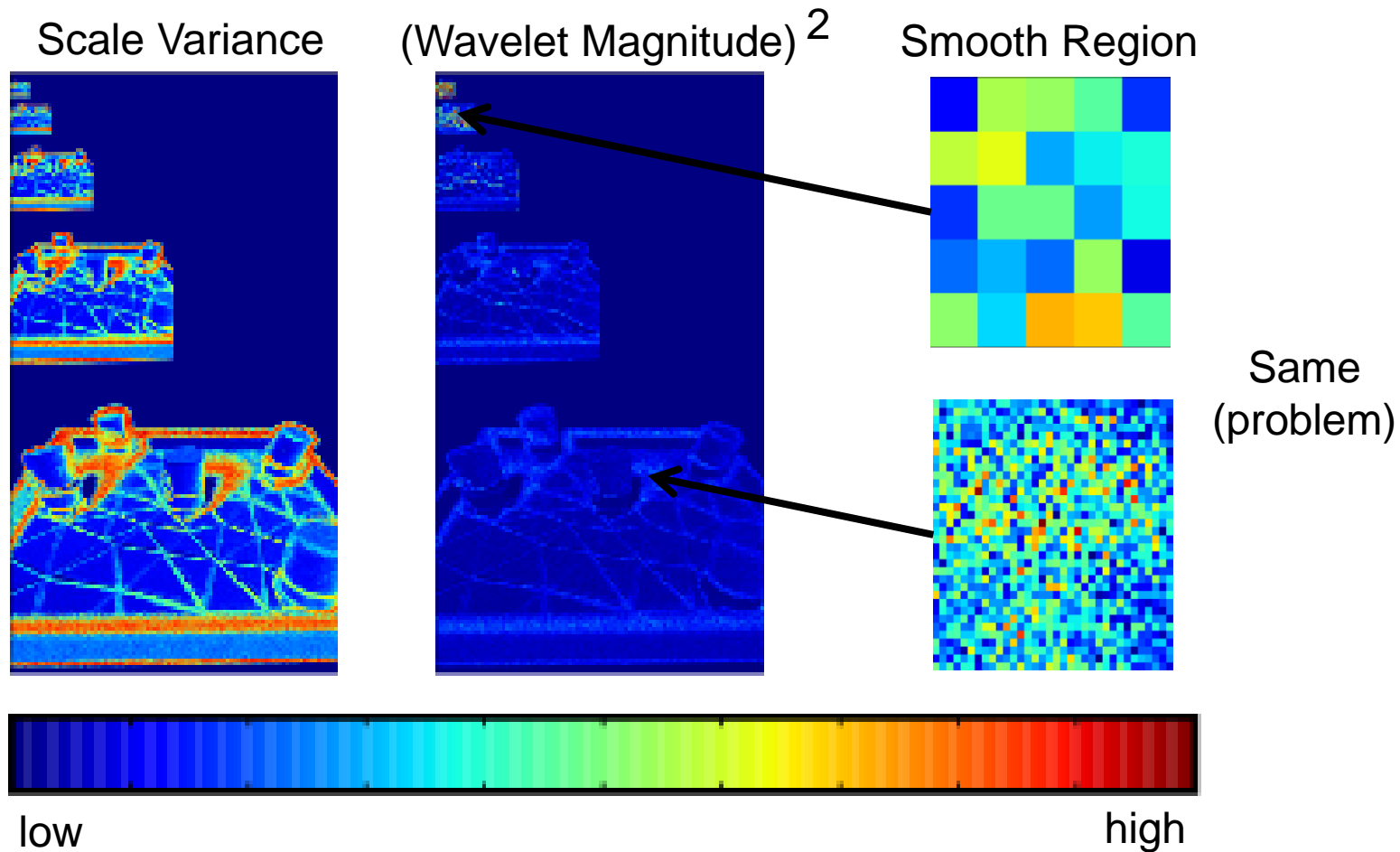
low

high

# Edge wavelets grow fine to coarse

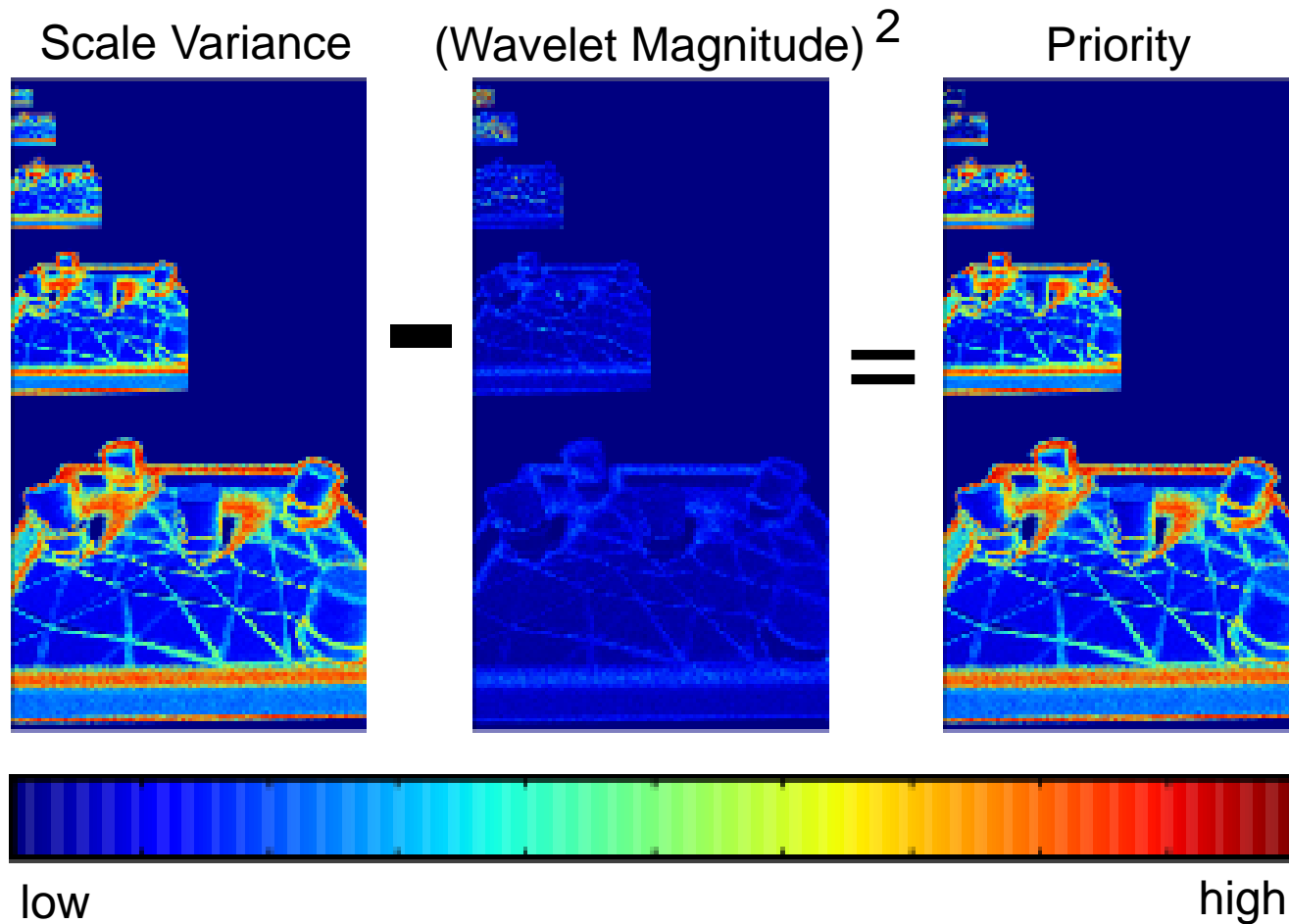


# Smooth wavelets stay the same

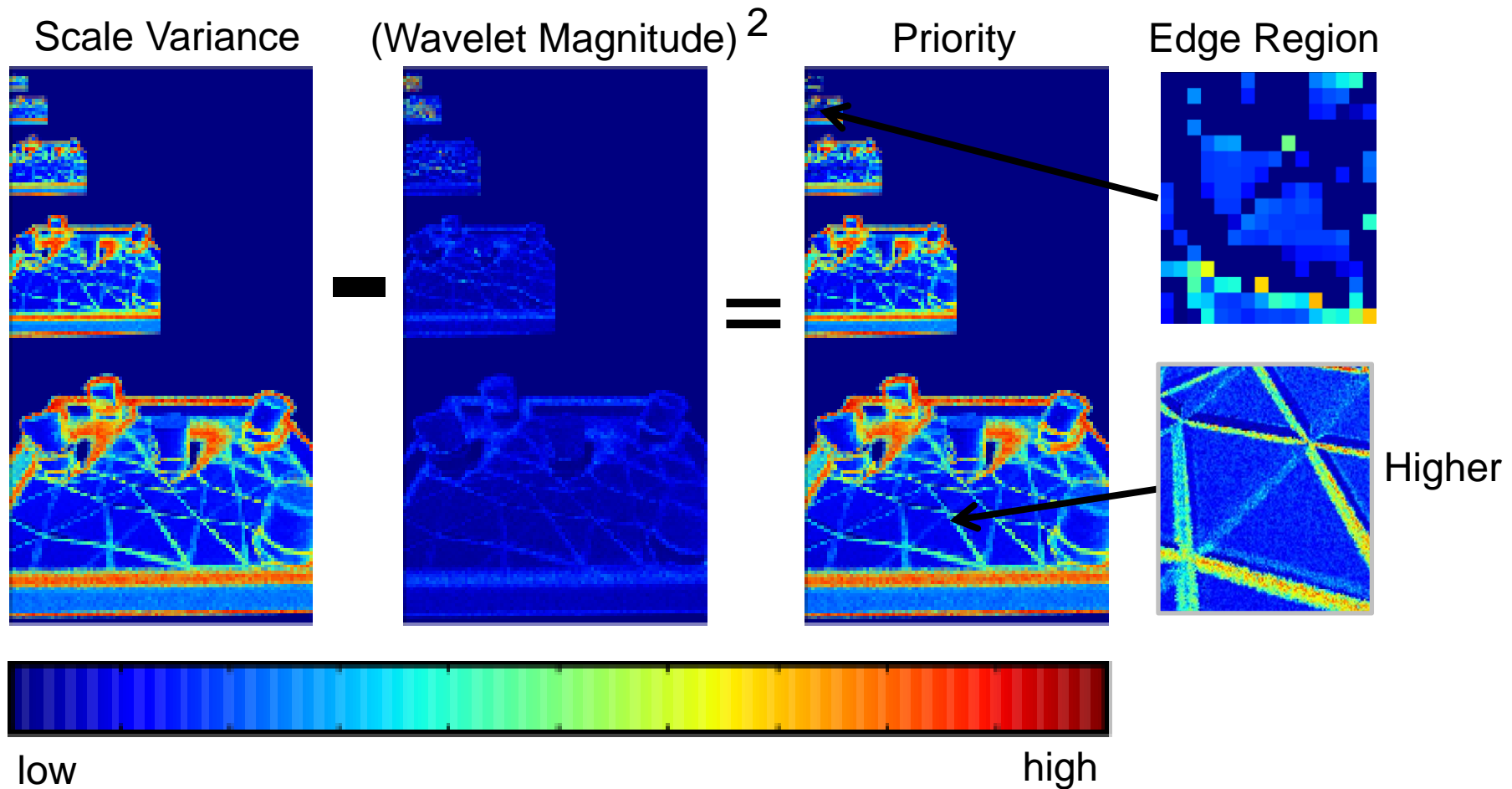




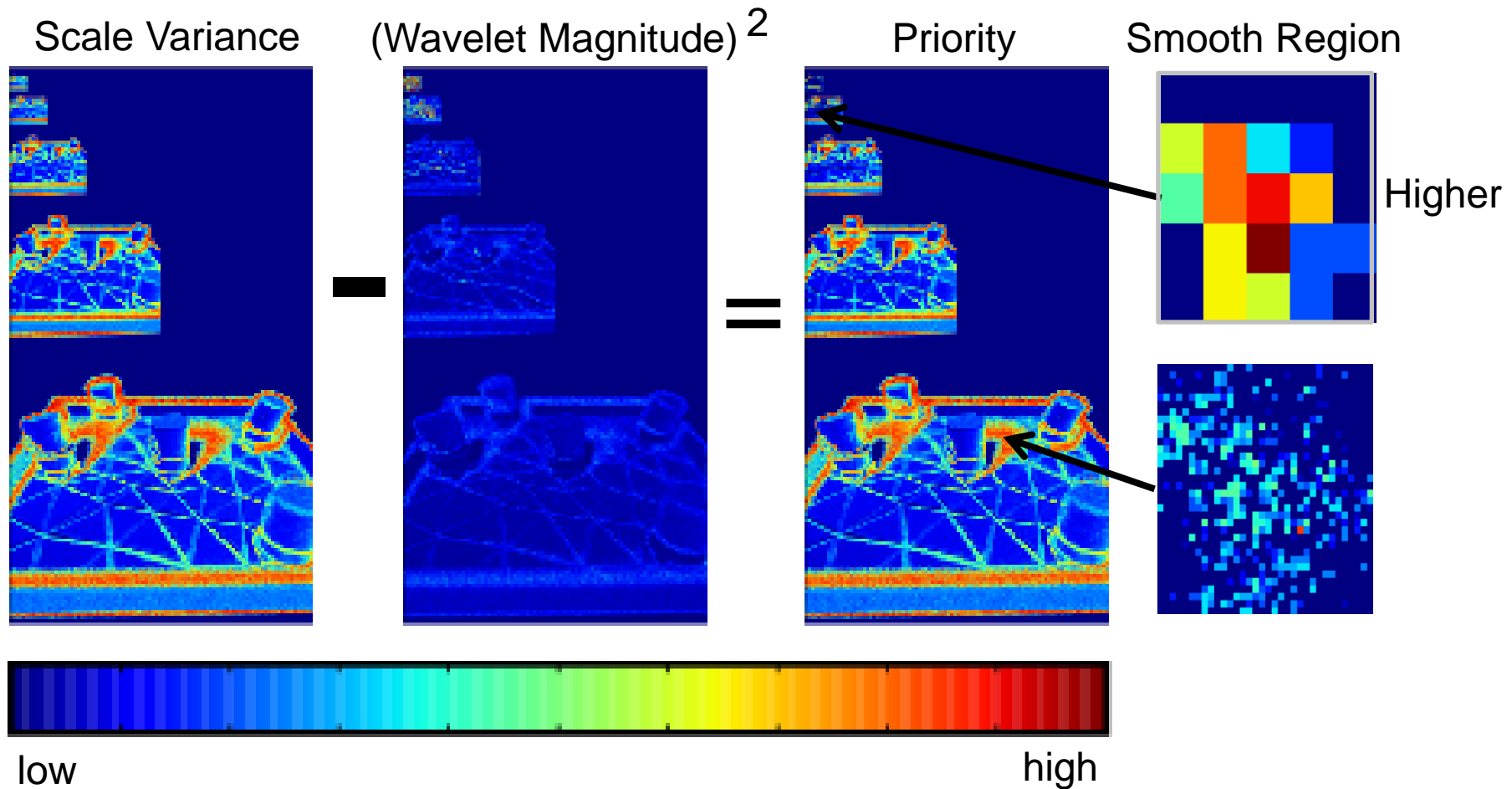
# Priority equals the difference



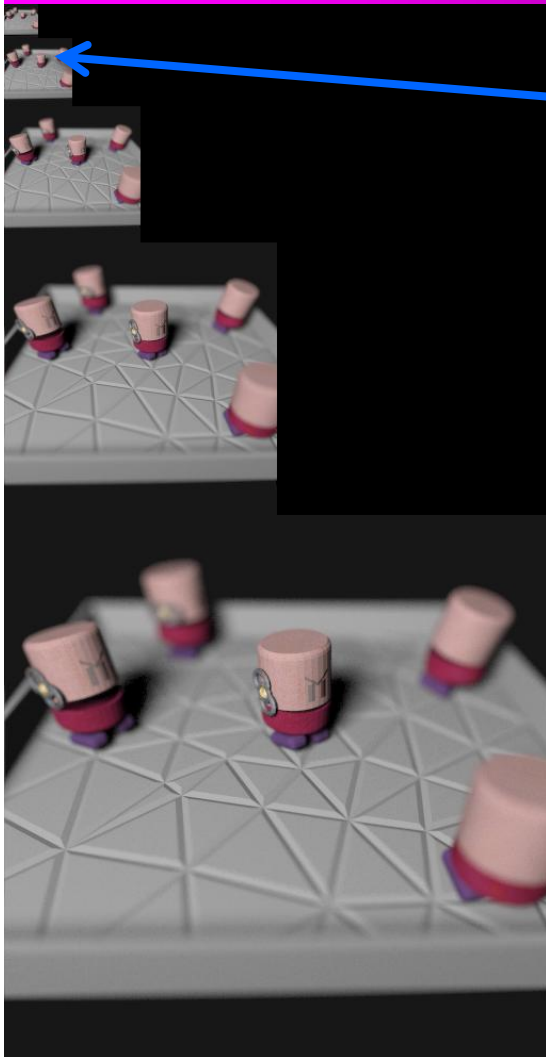
# Edges: Higher priority at fine scales



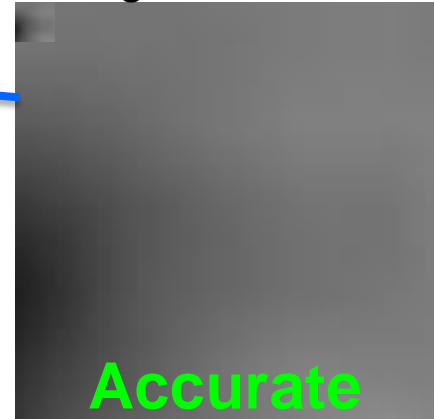
# Smooth: Higher priority at coarse scales



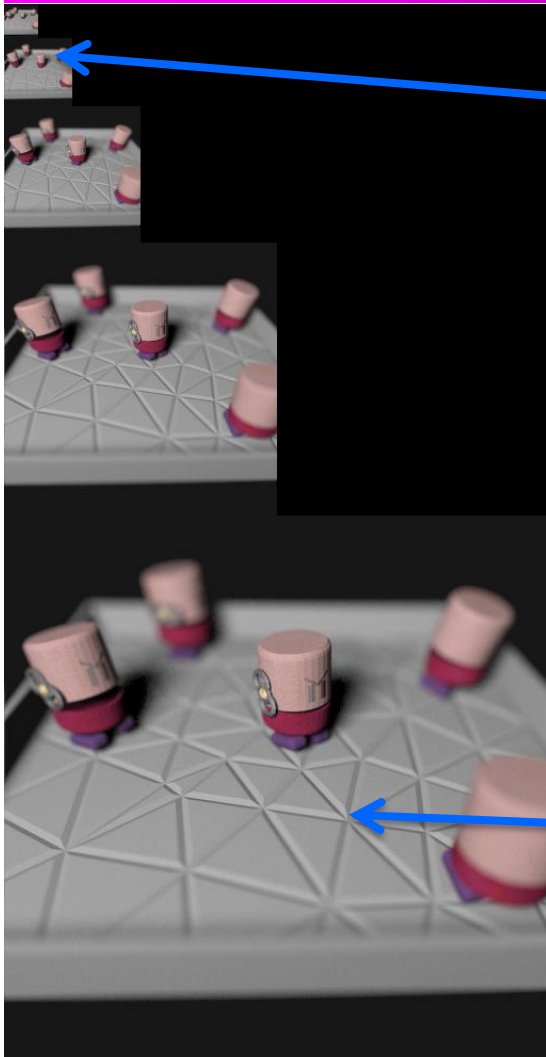
# After adaptive sampling



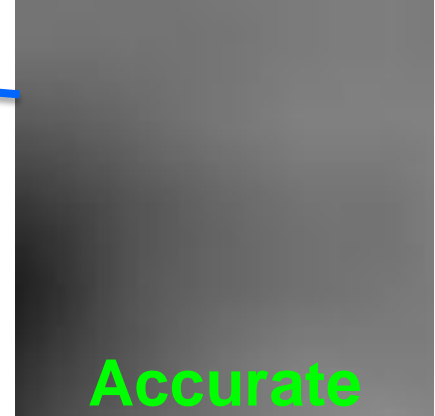
Smooth Region in Coarse Scale



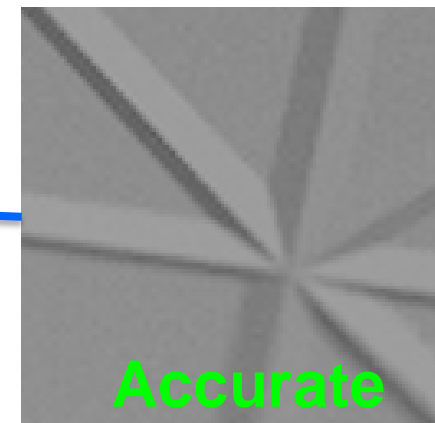
# After adaptive sampling



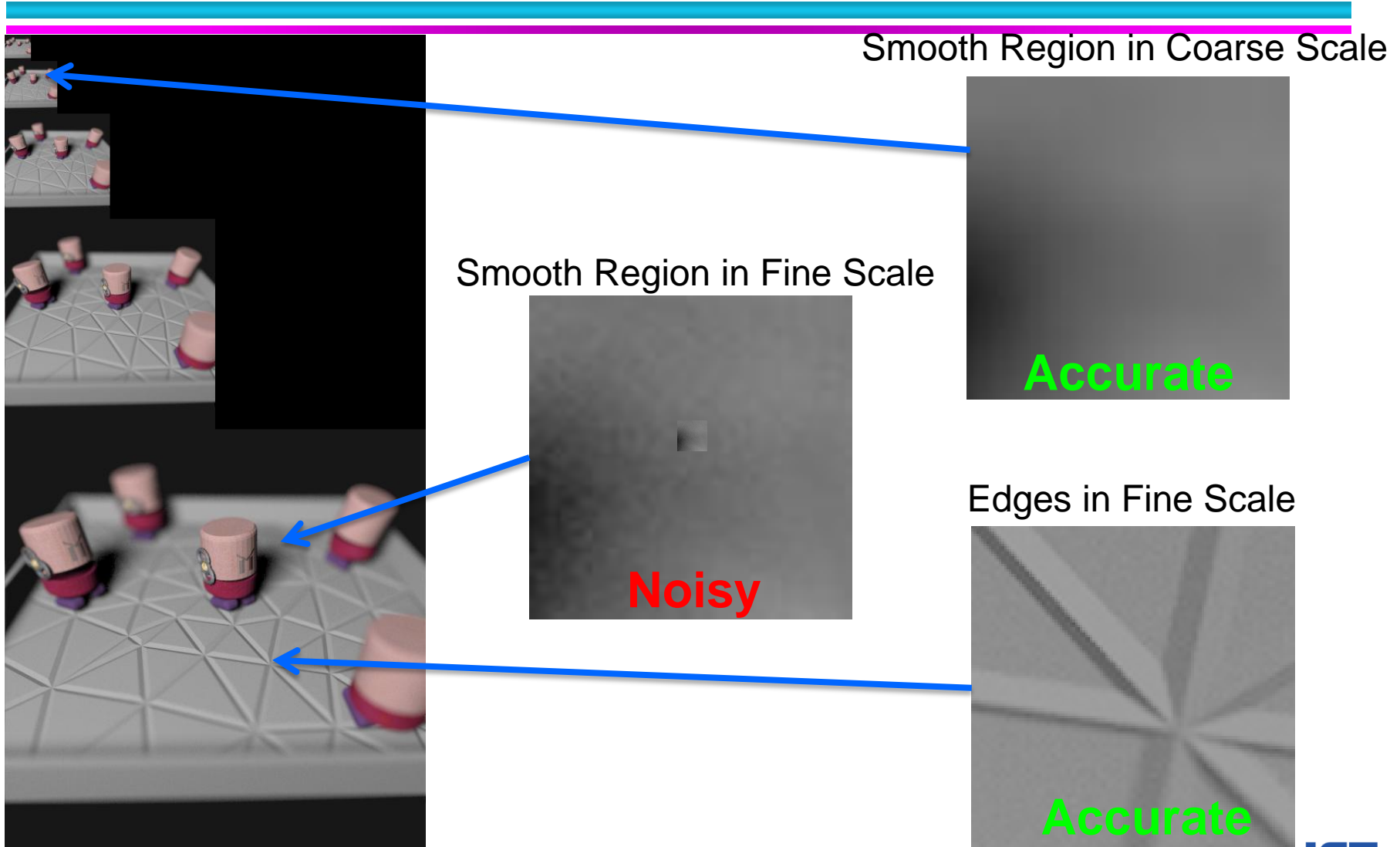
Smooth Region in Coarse Scale



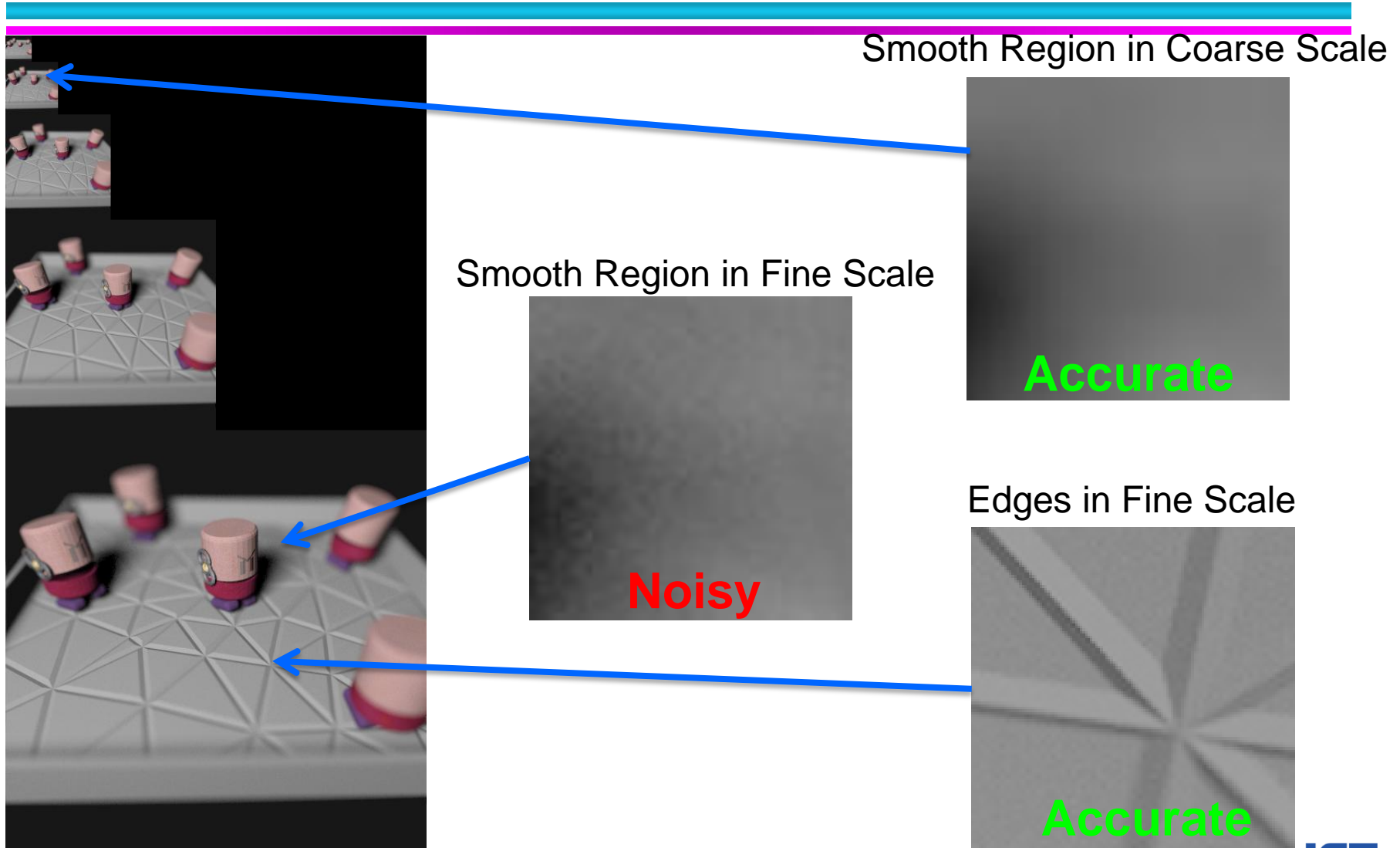
Edges in Fine Scale



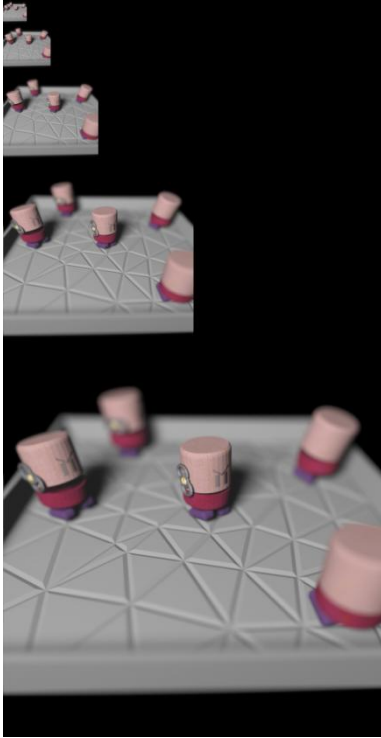
# After adaptive sampling



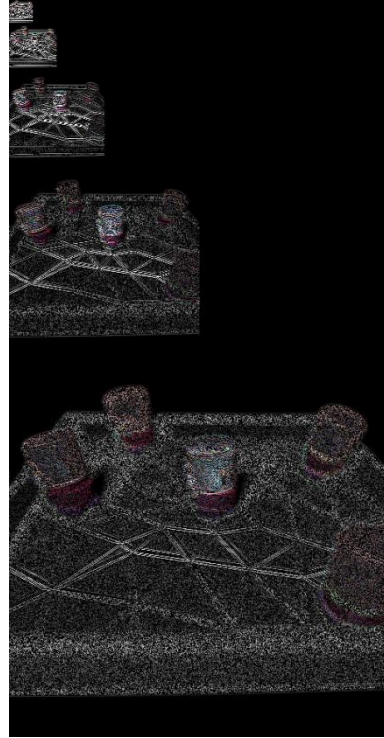
# Reconstruction: smooth away fine scale noise



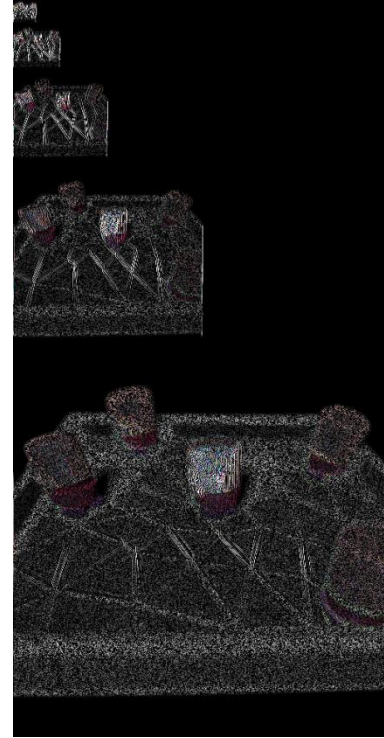
# Wavelets capture edges



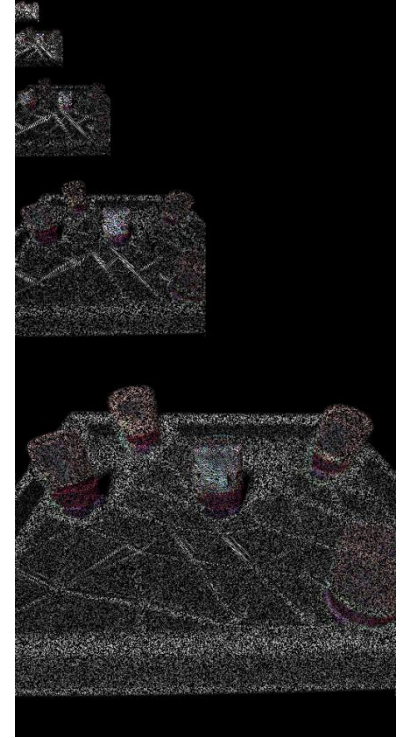
scale-scale



scale-wavelet



wavelet-scale

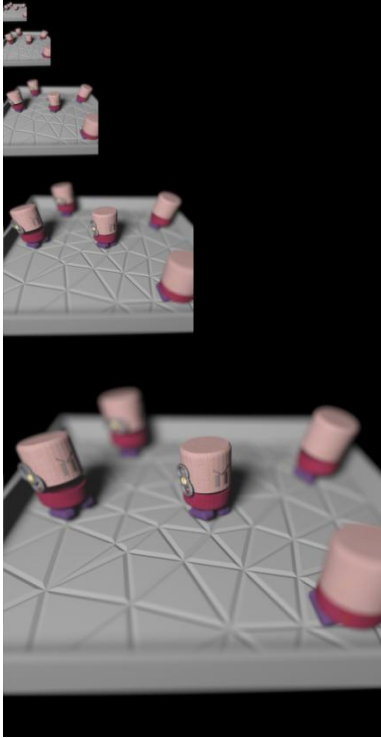


wavelet-wavelet

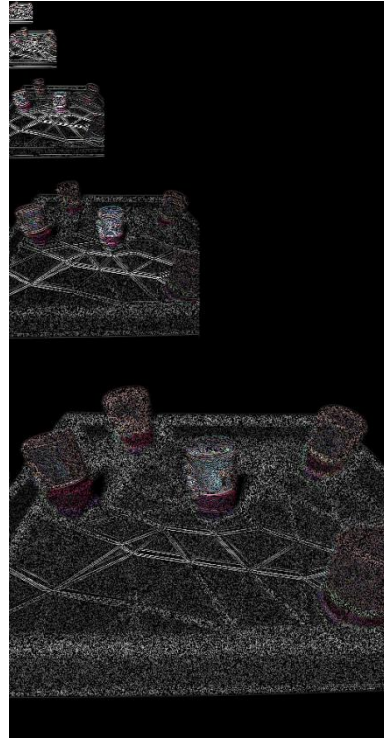
Edges



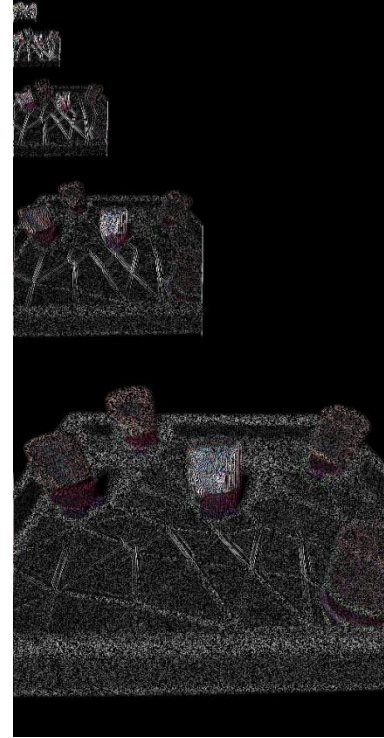
# Wavelets capture edges and Noise



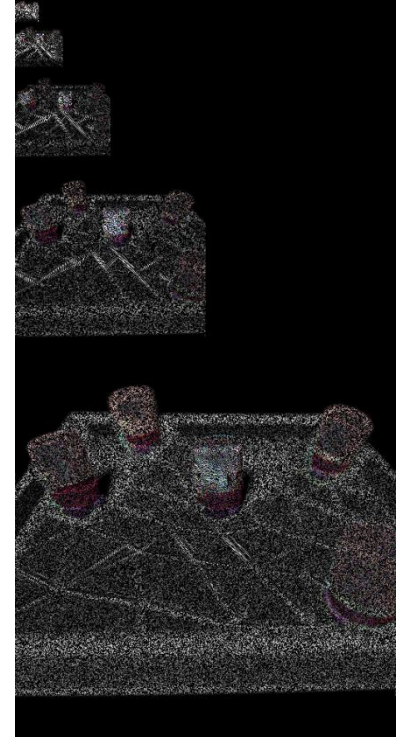
scale-scale



scale-wavelet



wavelet-scale



wavelet-wavelet

Edges and Noise

# Algorithm Outline

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**0) Start: 4 Samples per Pixel**

**1) Adaptive Sampling**

**-> 2) Reconstruction**

# Wavelet Reconstruction

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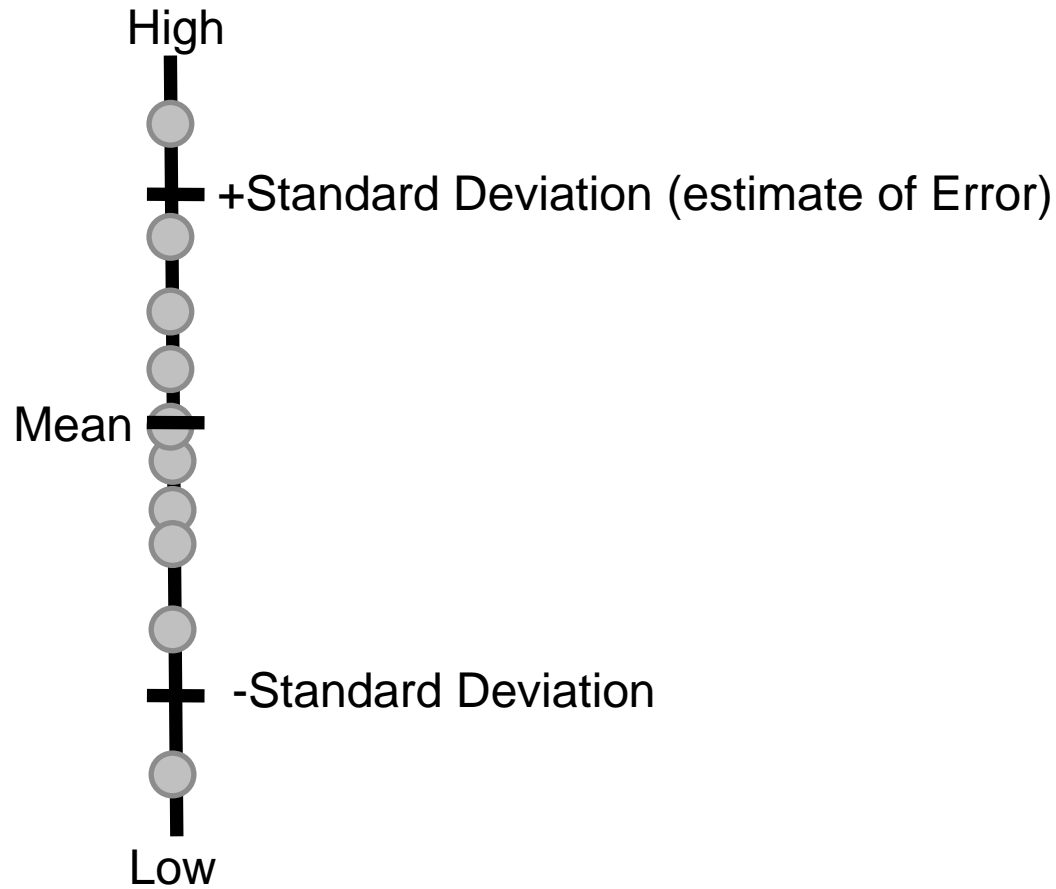
---

**Remove noise by suppressing wavelet magnitudes**

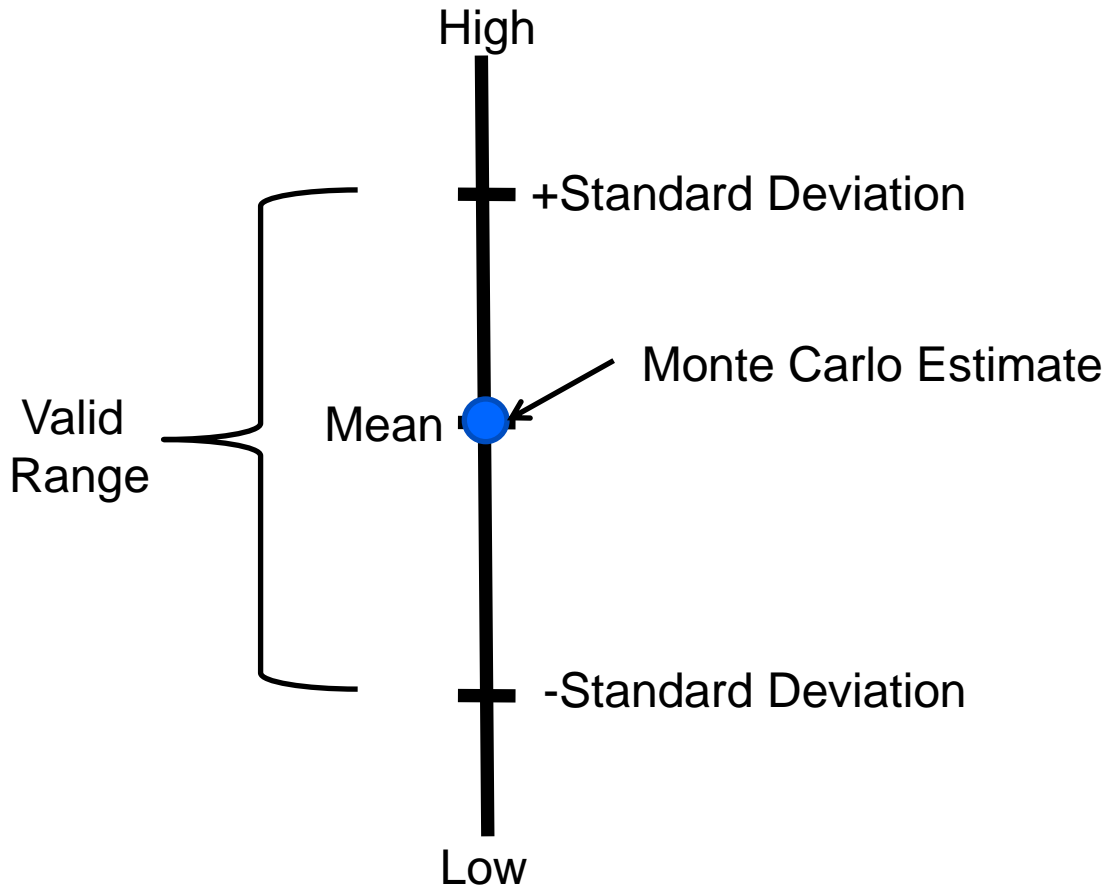
**How?**

**Choose smoothest image which fits samples**

# Monte Carlo: statistics



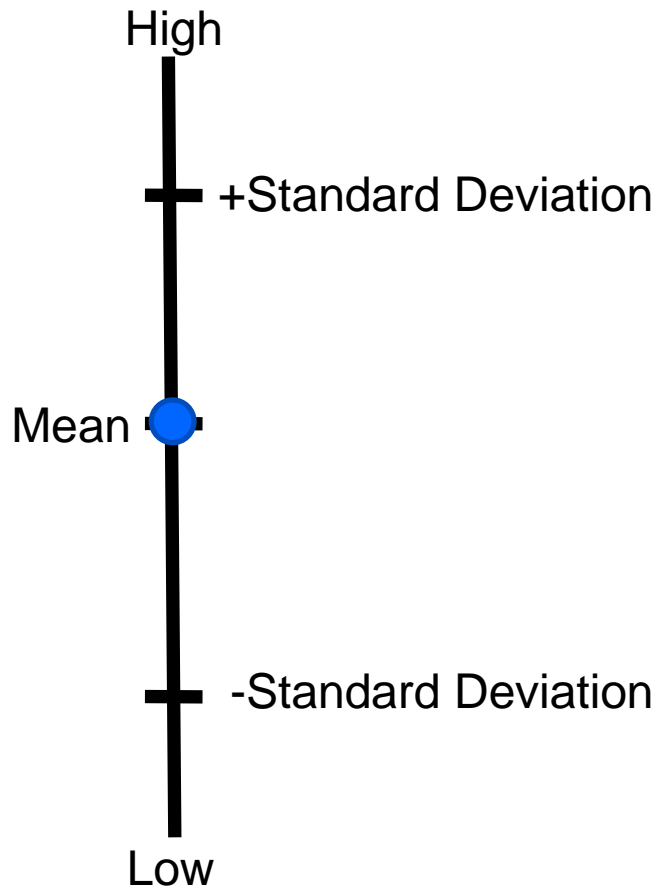
# Monte Carlo: statistics



# For pixel:

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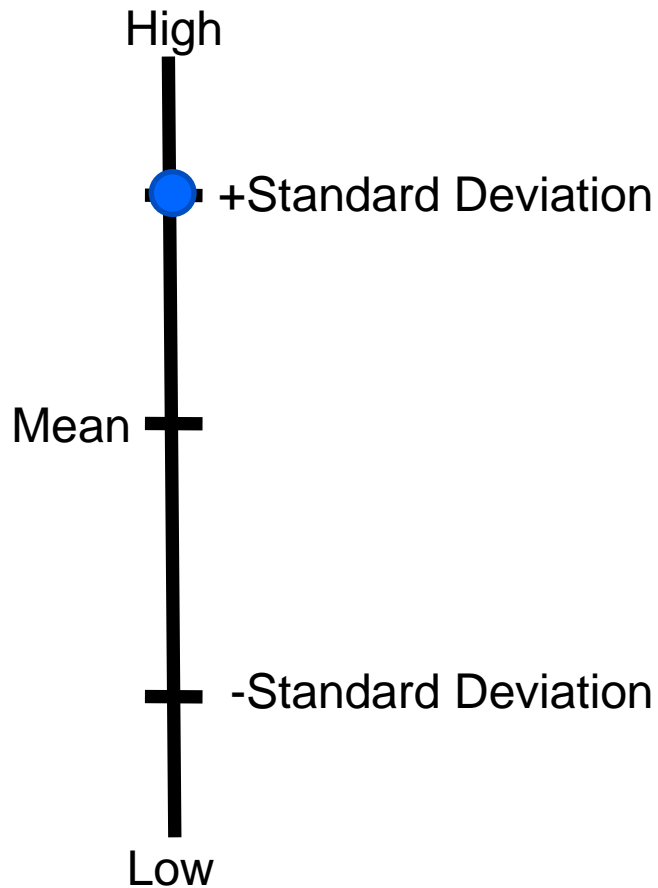
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# For pixel: luminance

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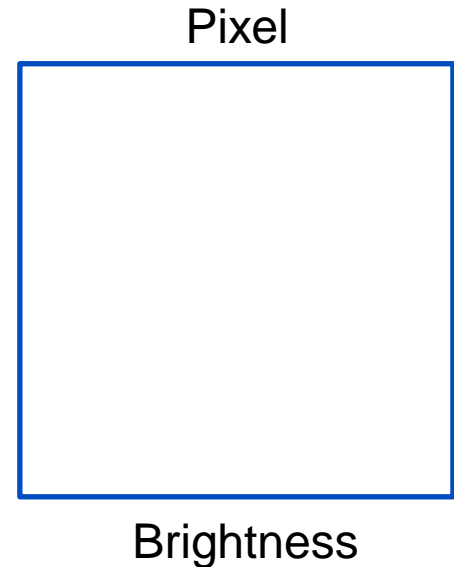
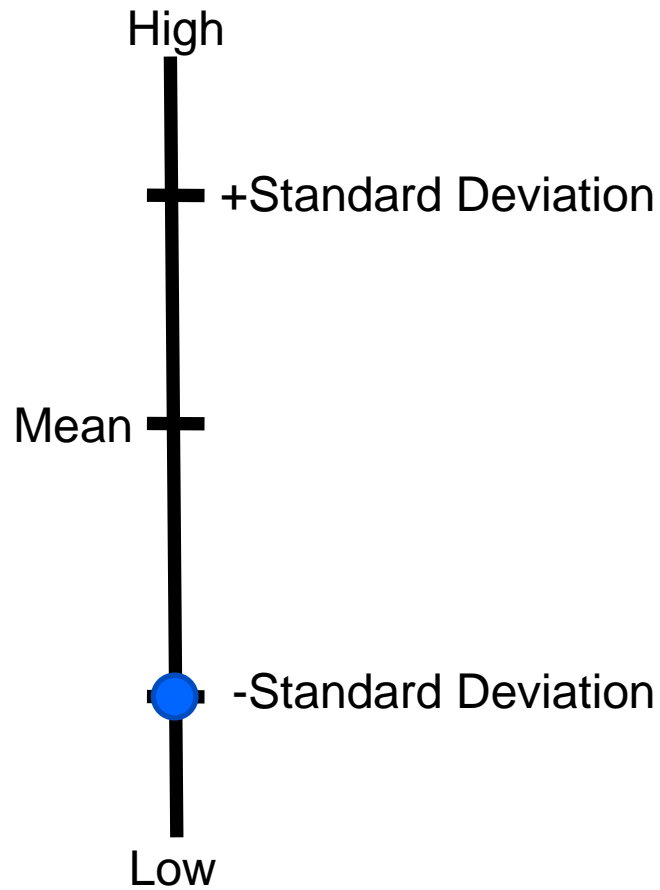
---



# For pixel: luminance

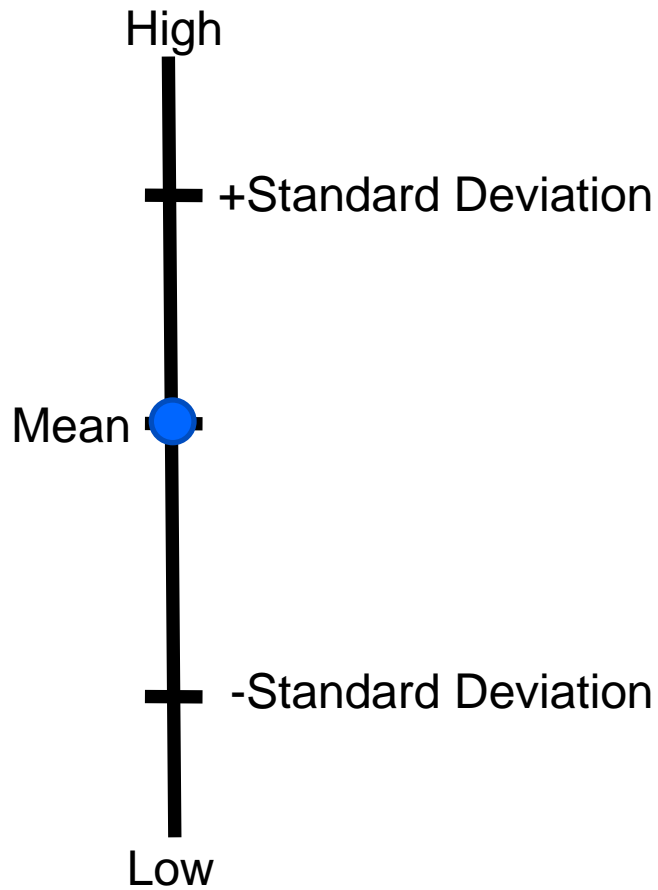
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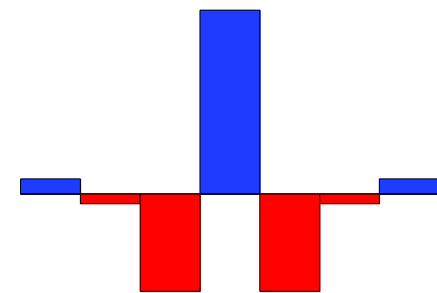




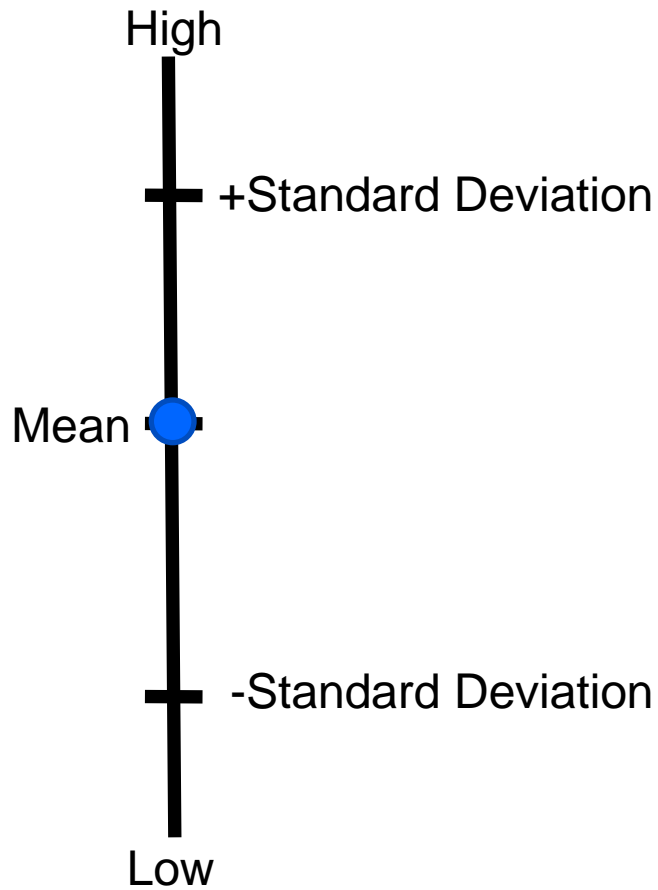
# For wavelet:



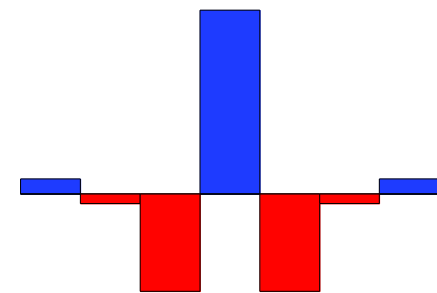
Wavelet Coefficient



# For wavelet: Smoothness

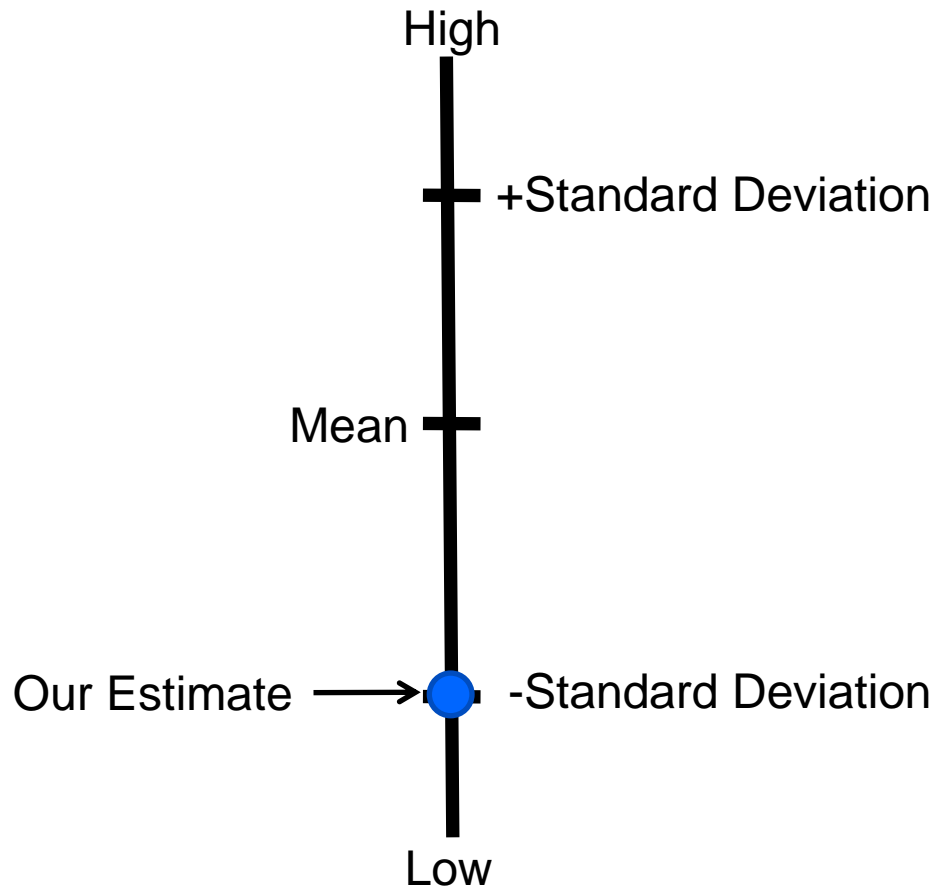


Wavelet Coefficient

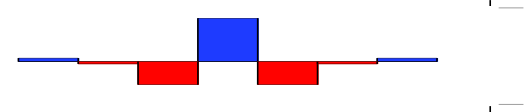


Smoothness

# Take the smoothest value



Wavelet Coefficient



Smoothness

# Reconstruction computation?!

standard deviation error

$$\Delta_{k,ij}^{\alpha} = \sqrt{\left\langle \sigma^2(\tilde{B}), \left(\Psi_{k,ij}^{\alpha}\right)^2 \right\rangle}.$$

Subtracting the standard deviation from the magnitude of the wavelet coefficients gives this result:

$$W_{k,ij}^{\alpha} = \text{sign}(\tilde{W}_{k,ij}^{\alpha}) \cdot \max\left(0, |\tilde{W}_{k,ij}^{\alpha}| - c_s \cdot \Delta_{k,ij}^{\alpha}\right), \quad (11)$$

where  $\tilde{W}$  are the wavelet coefficients from the pixel means,

$c_s$  (the smoothing constant) is a user-supplied constant

# After Sampling

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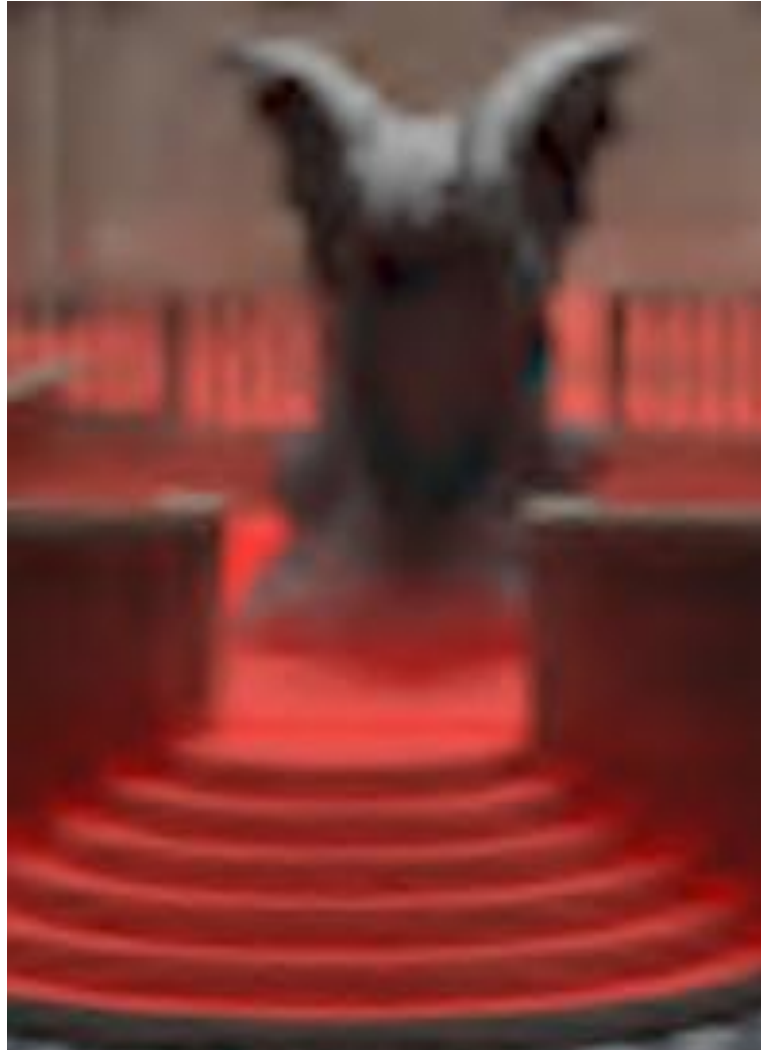
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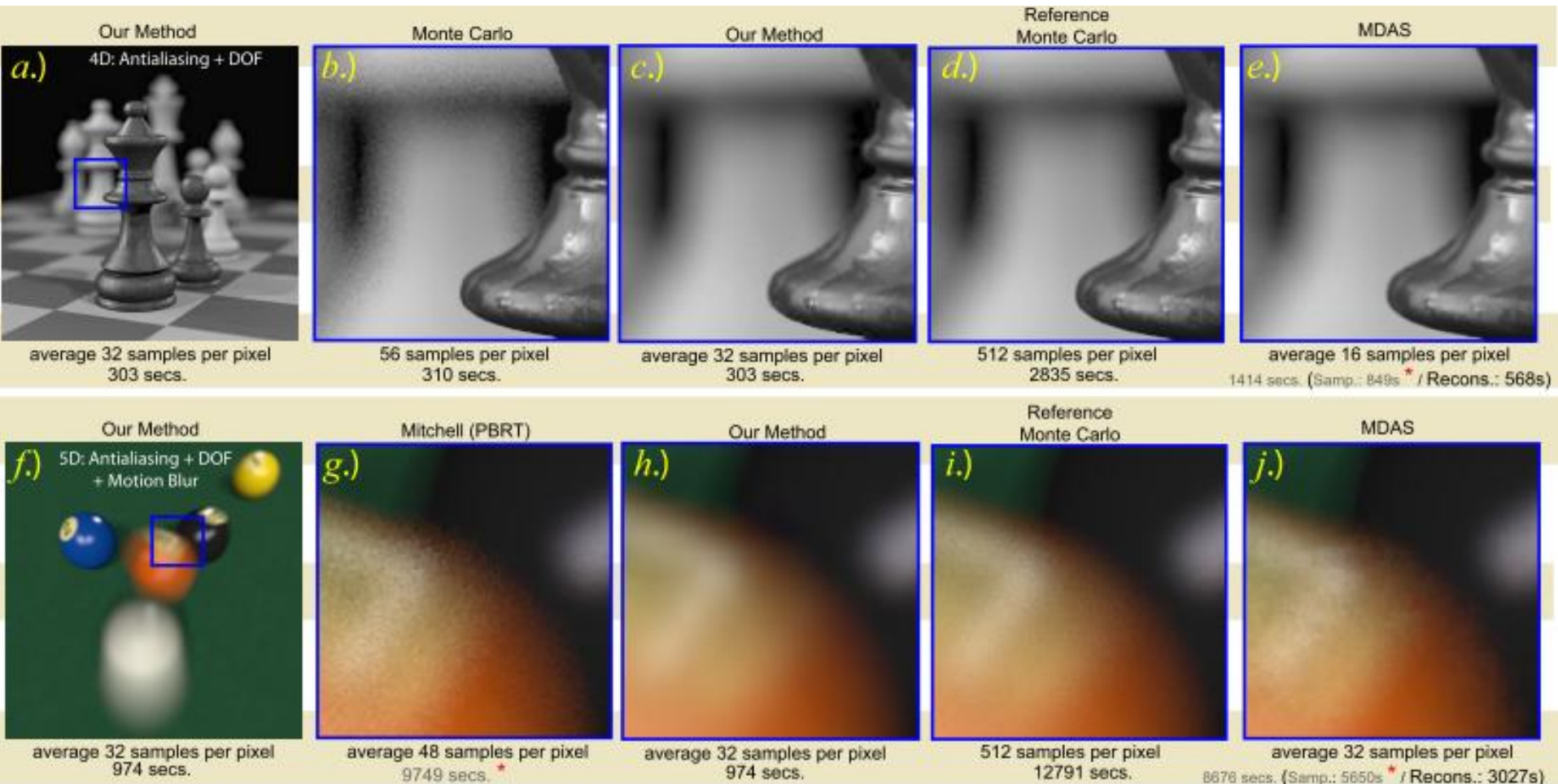
# After Reconstruction

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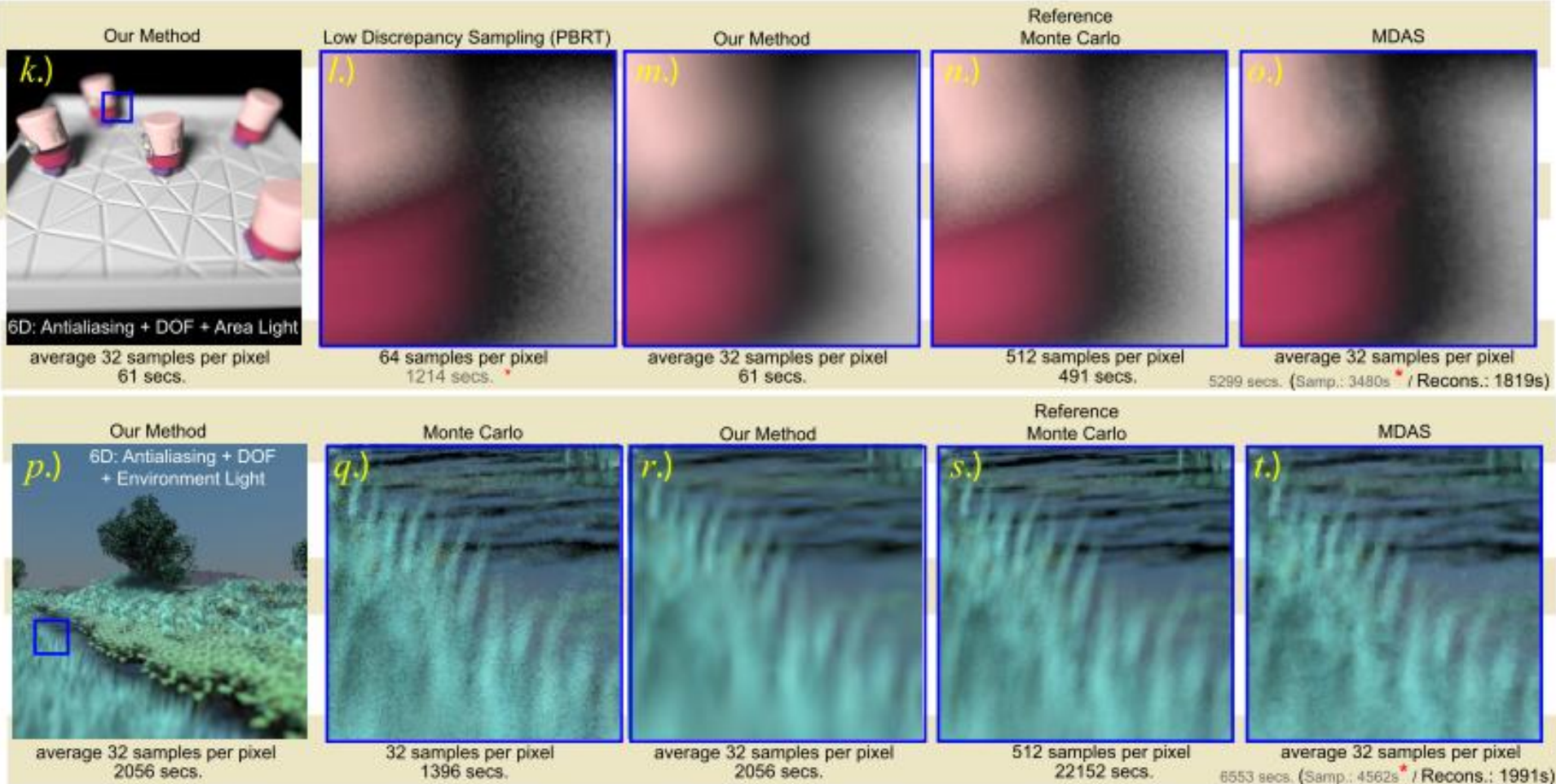
---



# Results (1/2)



# Results (2/2)

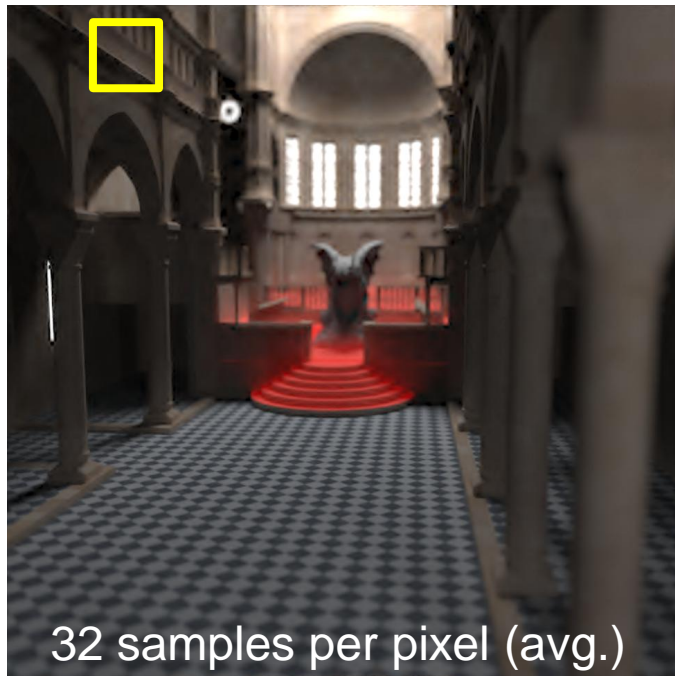




# Limitations (1/3)

## Wavelet artifacts when not enough samples

### Ringling around edges



Our Method

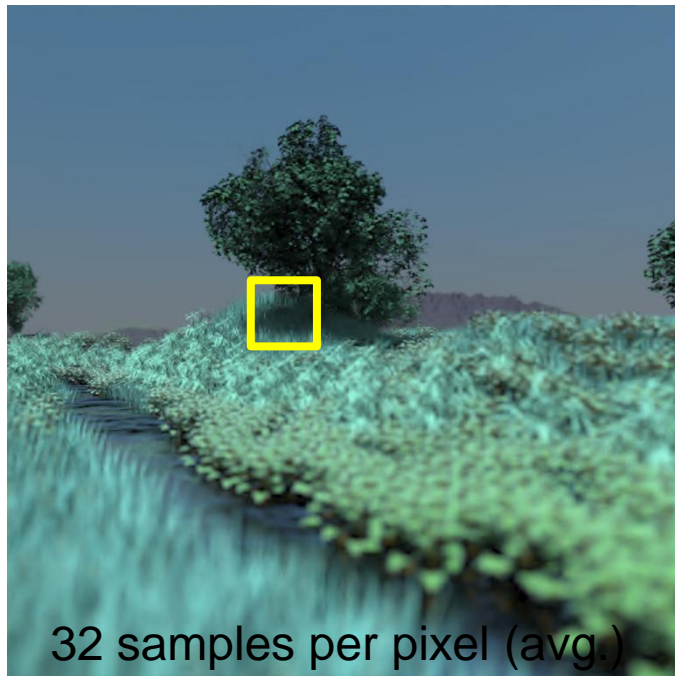


Monte Carlo

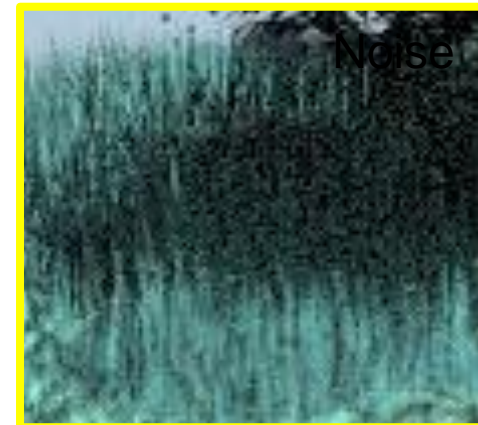
# Limitations (2/3)

## Wavelet artifacts when not enough samples

**Ringing around edges**  
**Overly smoothing**



Our Method



Monte Carlo

# Limitations (3/3)

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## **Wavelet artifacts when not enough samples**

**Ringling around edges**

**Overly smoothing**

## **Potential solutions**

**Variance reduction (path splitting, QMC, etc.)**

**Reduce smoothing during reconstruction**

**Use depth and normals to improve statistics**

**Use more samples**

# Conclusion/Summary

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**Sample and reconstruct in wavelet basis**

## **Features**

**Low Sample Counts**

**Efficient**

**General**

**Best for smooth image features**